PDHonline Course E104 (12 PDH)

Power Systems - Basic Concepts and Applications - Part I

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Power Systems - Basic Concepts and Applications

Part I

By Shih-Min Hsu, Ph.D., P.E.

MODULE 1: Introduction to Power Systems.

Overview

This module provides an introduction to power systems. It discusses a basic structure of power systems, the fundamentals of AC circuits, mathematical notations, balanced three-phase systems and per unit values.

Basic Structure of Power Systems

A power system is an interconnected network with components converting nonelectrical energy continuously into the electrical form and transporting the electrical energy from generating sources to the loads/users. A power system serves one important function and that is to supply customers with electricity as economically and as reliably as possible. It can be divided into three sub-systems:

Generation -	Generating and/or sources of electrical energy.
Transmission -	Transporting electrical energy from its sources to load centers with high voltages (115 kV and above) to reduce losses.
Distribution -	Distributing electrical energy from substations (44 kV ~ 12 kV) to end users/customers.

This basic structure of a power system is shown in Figure 1-1. The generator converts nonelectrical energy to electrical energy. The devices connecting generators to transmission

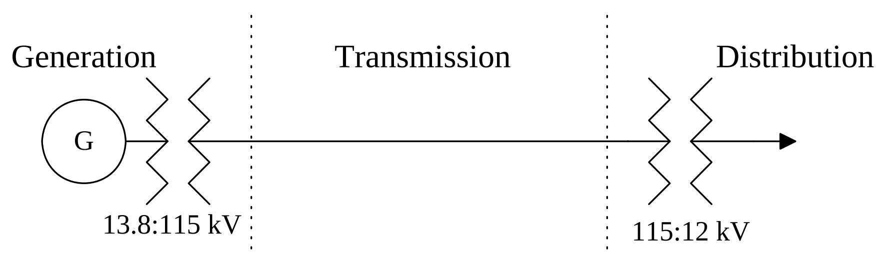

Fig. 1-1. A basic structure of a simplified power system.

system and from transmission system to distribution system are transformers. Their main functions are stepping up the lower generation voltage to the higher transmission voltage and stepping down the higher transmission voltage to the lower distribution voltage. The main advantage of having higher voltage in transmission system is to reduce the losses in the grid. Since transformers operate at constant power, when the voltage is higher, then the current has a lower value. Therefore, the losses, a function of the current square, will be lower at a higher voltage.

DC vs. AC

The current of a direct current (DC) circuit, as shown in Figure 1-2, consisting of a battery and a pure resistive load can be calculated as

$$I_{DC} = \frac{V_{DC}}{R},$$

and the power provided by the voltage source equals

$$P_{DC} = V_{DC} I_{DC} = \frac{V_{DC}^2}{R} = I_{DC}^2 R,$$

where

I_{DC} = DC current in Amperes (A),
V_{DC} = DC voltage in Volts (V),
P_{DC} = DC power in Watts (W),

and

R = the load resistance in Ohms (Ω).

The voltage and current waveforms are shown in Figure 1-3.

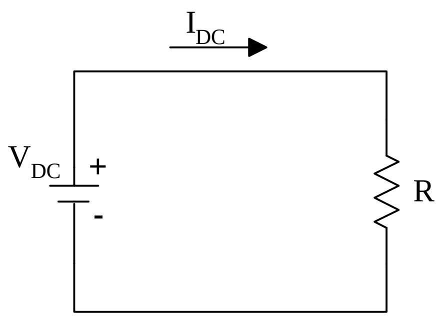

Fig. 1-2. A simple DC circuit.

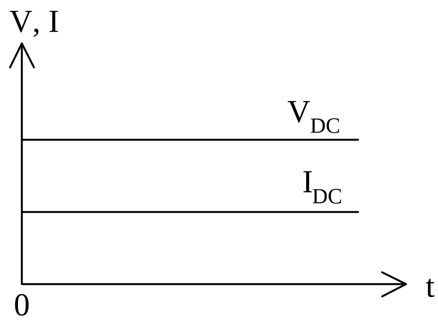

Fig. 1-3. Voltage and current waveforms of the simple DC circuit.

Example 1-1: A DC circuit, as shown in Figure 1-2, has a DC voltage of 12 volts and a resistor of 2 Ω. What are the DC current in the circuit and the power consumed by the resistor?
Solution:

$$I_{DC} = \frac{12}{2} = 6 \quad (A)$$

$$P_{DC} = 12 \times 6 = 72 \quad (W)$$ ♦

It is worth mentioning that these DC quantities are real numbers not complex numbers.

There is another category of circuits, the alternating current (AC) circuits. Since in power systems the sinusoidal voltages are generated, and consequently, most likely sinusoidal currents are flowed in the generation, transmission and distribution systems, sinusoidal quantities are assumed throughout this material, unless otherwise specified.

In general, a set of typical steady-state voltage and current waveforms of an AC circuit can be drawn as shown in Figure 1-4, and their mathematical expressions can be written as follows:

$$v(t) = V_m \cos(\omega t),$$

and

$$i(t) = I_m \cos(\omega t + \theta),$$

where

V_m, I_m = the peak or the maximum values of the voltage and current waveforms,
ω = angular frequency in radians/second,

and

θ = phase angle with respect to the reference in degrees or in radians.

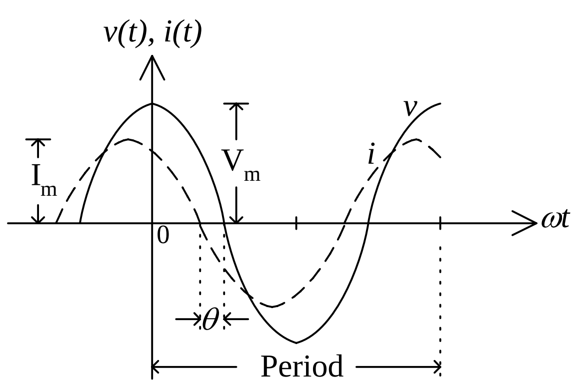

Fig. 1-4. Typical voltage and current waveforms

The period in Figure 1-4 can be 360° or 2π. In some cases, the period can be in time, for instance, 0.016667 second for 60 Hz.

There is an important quantity called "root mean square" value, or rms, and is defined as

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}\,.$$

For a sinusoidal voltage, its rms value equals

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T [V_m \cos(\omega t)]^2 dt} = \sqrt{\frac{V_m^2}{\omega T}\int_0^{2\pi} \frac{1+\cos 2\omega t}{2} d(\omega t)} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\omega t}{2} + \frac{\sin 2\omega t}{4}\right]_0^{2\pi}} = \frac{V_m}{\sqrt{2}}\,.$$

For example, a typical household voltage of 120 volts is rms value, and its peak value is $120 \times \sqrt{2} = 170$ volts. Since the frequency in U.S. is 60 Hertz (Hz), the angular frequency $\omega = 2 \times \pi \times 60 = 377$. Such a voltage can be expressed as

$$v(t) = 120\sqrt{2}\cos(377t) = 170\cos(377t) \text{ volts.}$$

If the current lags the voltage by 30° and its magnitude is a half of the voltage, then it can be written as follows

$$i(t) = 85\cos(377t - 30°) \text{ amps.}$$

Phasor Representations

It may not be convenient to express the voltages and currents in instantaneous forms all the time. As utilized in AC circuits, phasor representations are used in power systems because of convenience.

Recall Euler's identity

$$e^{j\theta} = \cos\theta + j\sin\theta\,.$$

Then, the current can be re-written as

$$i(t) = \text{Re}[I_m \cos(\omega t + \theta) + jI_m \sin(\omega t + \theta)] = \sqrt{2}\,\text{Re}(\frac{I_m}{\sqrt{2}} e^{j\theta} \cdot e^{j\omega t}) = \sqrt{2}\,\text{Re}(\bar{I} e^{j\omega t})\,,$$

where $\bar{I}$ is defined as the phasor (or polar) representation of *i*(*t*), and is a complex number in general. It has two parts, the magnitude and phase angle, namely,

$$\bar{I} = |\bar{I}|\angle\bar{I} = \frac{I_m}{\sqrt{2}}\angle\theta = I\angle\theta\,,$$

where I is the rms value of the current, and the subscript "rms" is commonly neglected. Figure 1-5 shows the graphical representation of $\bar{I}$. The direction of the phase angle is defined as counterclockwise for a positive value, and clockwise for a negative value.

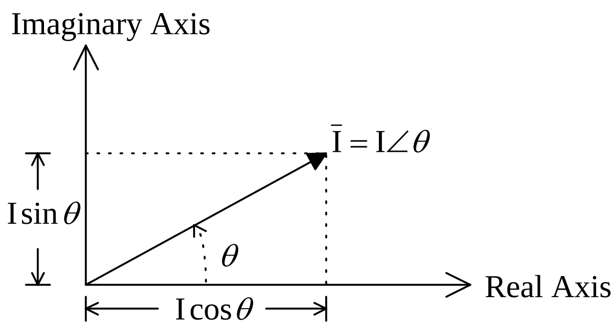

Fig. 1-5. A graphical representation of $\bar{I}$.

From Figure 1-5, it is obvious that a phasor can be expressed not only in a "polar form" but also in a "rectangular form", namely,

$$\bar{I} = I\angle\theta = I\cos\theta + jI\sin\theta \, .$$

One of the advantages of using phasor representations instead of instantaneous forms is that one can add sinusoidal functions of the same frequency by expressing them as phasors and then adding the phasors by the rules of vector algebra. More information on the phasor (or polar) representation can be found in the Appendix 1A.

Example 1-2: What are the phasor representations of the following instantaneous quantities?

$v(t) = 170\cos(377t)$ volts, and $i(t) = 85\cos(377t + 30°)$ amps

Solution:

$$\bar{V} = \frac{170}{\sqrt{2}}\angle 0° = 120\angle 0° \text{ volts}$$

$$\bar{I} = \frac{85}{\sqrt{2}}\angle 30° = 60.1\angle 30° \text{ amps}$$ ♦

Unlike in DC circuits, the loads in AC circuits, as shown in Figure 1-6, can be expressed as its impedance, consisting of resistance R and reactance X, as follows

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = R + (j\omega L + \frac{1}{j\omega C}) = R + j(X_L - X_C) = R + jX = Z\angle\theta \ \Omega,$$

where

$$Z = \sqrt{R^2 + X^2}\,,$$

and

$$\theta = \tan^{-1}(\frac{X}{R})\,.$$

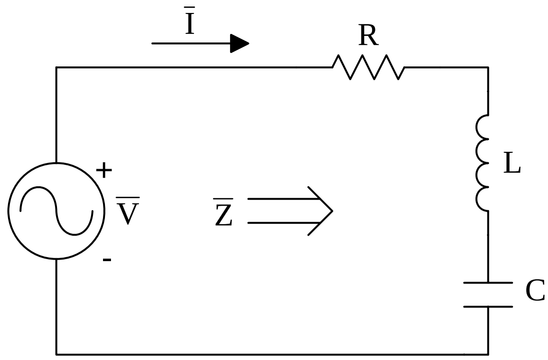

Fig. 1-6. RLC circuit.

Example 1-3: A 60 Hz 120 volts AC voltage source is connected to a 10Ω resistor, a 31.83 mH inductor and 1326.26μF capacitor, as shown in Figure 1-6.
Find

(1) The total impedance $\bar{Z}$.
(2) The current $\bar{I}$ in polar form.
(3) The voltage and current in instantaneous forms.

Solution:
(1) Since the frequency is 60 Hz, the inductive and capacitive reactances can be obtained as

$$X_L = \omega L = 377 \times 31.83 \times 10^{-3} = 12\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{377 \times 1326.26 \times 10^{-6}} = 2\Omega$$

The total impedance seen by the voltage source

$$\bar{Z} = R + j(X_L - X_C) = 10 + j(12 - 2) = 10 + j10 = \left(\sqrt{10^2 + 10^2}\right)\angle \tan^{-1}\left(\frac{10}{10}\right) = 10\sqrt{2}\angle 45° \, \Omega$$

(2) To calculate the current, the angle of the voltage is set to be the reference, namely, 0°. Then,

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{120\angle 0°}{10\sqrt{2}\angle 45°} = 8.485\angle -45° \text{ amps}$$

(3) To convert the phasors to the instantaneous forms

$$v(t) = 120\sqrt{2}\cos(377t) = 170\cos(377t) \text{ volts}$$

$$i(t) = 8.485\sqrt{2}\cos(377t - 45°) = 12\cos(377t - 45°) \text{ amps}$$ ♦

When the imaginary part of the impedance is positive, the load is called an inductive load, and the current lags the voltage. On the other hand, if the imaginary part of the impedance is negative, the load is called a capacitive load, and the current leads the voltage.

An admittance is the inverse of its impedance and can be expressed as

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{\bar{I}}{\overline{V}} = G + jB,$$

where G is the conductance and B is the susceptance, both in siemen, S.

Power in Sinusoidal Steady-State Conditions

If the voltage and the current of an AC circuit, as shown in Figure 1-7, are expressed in their instantaneous forms as follows

$$v(t) = \sqrt{2}V\cos(\omega t - \theta_v),$$

and

$$i(t) = \sqrt{2}I\cos(\omega t - \theta_i).$$

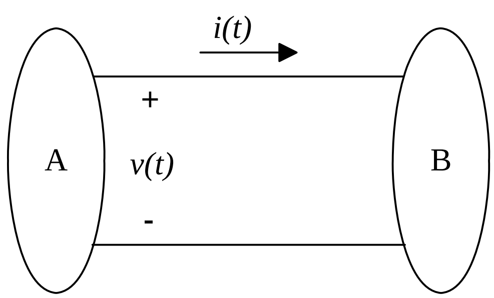

Fig. 1-7. Power transfer between two systems.

Then, the instantaneous power from A to B is the product of *v(t)* and *i(t)* as follows

$$p(t) = v(t)i(t) = 2VI[\cos(2\omega t - \theta_v - \theta_i) + \cos(\theta_v - \theta_i)].$$

The real power or the active power is defined as the average of the instantaneous power in one period

$$P = \frac{1}{T}\int_0^T p(t)dt = VI\cos\phi \quad \text{(W)},$$

where the angle $\phi = \theta_v - \theta_i$, and is defined as the phase angle of the current "lags" the voltage. The complex power $\overline{S}$ from A to B can be calculated as

$$\overline{S} = \overline{V}\overline{I}^* = (V\angle\theta_v)(I\angle\theta_i)^* = VI\angle\phi = VI\cos\phi + jVI\sin\phi = P + jQ \qquad \text{(VA)},$$

where

$P = \mathrm{Re}\{\overline{S}\} = VI\cos\phi$, is the active power in watts (W),

and

$Q = \mathrm{Im}\{\overline{S}\} = VI\sin\phi$, is the reactive power in voltamperes reactive (var).

Furthermore, can be expressed in Polar form as follows

$$\overline{S} = \left(\sqrt{P^2 + Q^2}\right)\angle\tan^{-1}\left(\frac{Q}{P}\right) = \left|\overline{S}\right|\angle\phi .$$

The magnitude of the complex power is defined as the apparent power

$$S = \left|\overline{S}\right| = VI = \sqrt{P^2 + Q^2} \qquad \text{(VA).}$$

A graphical presentation of complex power, active power and reactive power is called a power triangle (or P-Q triangle), as shown in Figure 1-8. It is worth mentioning that these power quantities are directional. For instance, when P is positive, the power flow is as defined, from A to B. If the P is negative, the power flow is actually from B to A.

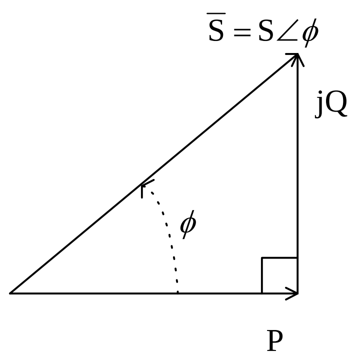

Fig. 1-8. Power (P-Q) triangle.

The power factor, pf, is defined as

$$pf = \frac{P}{S} = \frac{VI\cos\phi}{VI} = \cos\phi .$$

Therefore, the phase angle ϕ is sometimes called the power factor angle. A lagging power factor indicates an inductive impedance and therefore a positive value for ϕ. Similarly, a leading power factor implies a capacitive impedance, and therefore a negative value for ϕ.

Single-Phase and Balanced Three-Phase Systems

The above-mentioned concepts are intended for single-phase systems. However, electric power is generated, transported and distributed mainly in a symmetrical (balanced) three-phase structure. In particular, the power is generated by three-phase synchronous generators. It is then transmitted and distributed in the form of three-phase power, except for the lowest voltage levels of distribution system where single-phase systems are used. Symmetry is one of the most important characteristic features of the three-phase systems. This is because three-phase symmetrical systems are more effective than other ones, in particular, single-phase systems with respect to the capability of power transmission.

In general, the phase voltages of a balanced three-phase voltage source with "positive" sequence can be expressed as

$$v_{an}(t) = \sqrt{2}\mathrm{V}\cos(\omega t),$$

$$v_{bn}(t) = v_{an}(t - \frac{\mathrm{T}}{3}) = \sqrt{2}\mathrm{V}\cos(\omega t - 120°),$$

$$v_{cn}(t) = v_{an}(t - \frac{2\mathrm{T}}{3}) = \sqrt{2}\mathrm{V}\cos(\omega t - 240°) = \sqrt{2}\mathrm{V}\cos(\omega t + 120°).$$

Their waveforms are shown in Figure 1-9. Their phasor representations are obtained as discussed for single-phase systems

$$\overline{\mathrm{V}}_{\mathrm{an}} = \mathrm{V}\angle 0°,$$

$$\overline{\mathrm{V}}_{\mathrm{bn}} = \overline{\mathrm{V}}_{\mathrm{an}}\angle -120° = \mathrm{V}\angle -120°,$$

$$\overline{\mathrm{V}}_{\mathrm{cn}} = \overline{\mathrm{V}}_{\mathrm{an}}\angle 120° = \mathrm{V}\angle 120°.$$

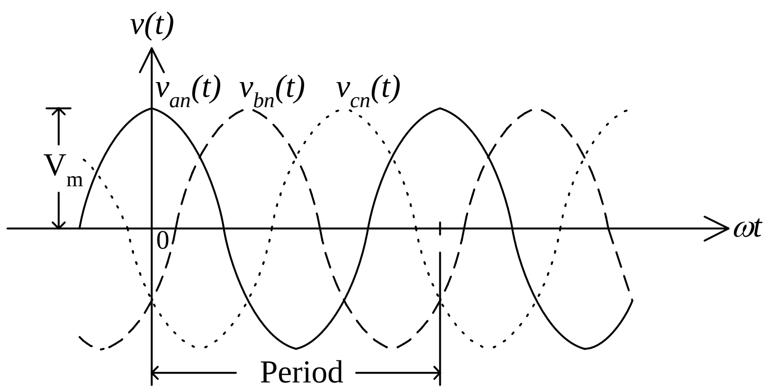

Fig. 1-9. Waveforms of phase voltages of balanced three-phase systems.

The above notation is obtained with the assumption that the symmetrical voltage source has a "positive" (or a-b-c) sequence. If a "negative" (or a-c-b) sequence is assumed, then $\overline{\mathrm{V}}_{\mathrm{bn}}$ leads $\overline{\mathrm{V}}_{\mathrm{an}}$ by 120° and $\overline{\mathrm{V}}_{\mathrm{cn}}$ lags $\overline{\mathrm{V}}_{\mathrm{an}}$ by 120°.

Three-phase voltage sources are generally connected in two different configurations, namely, a wye connection and a delta connection. An ideal three-phase voltage source with a wye connection is shown in Figure 1-10, and its line-to-line voltage, for instance $\overline{\mathrm{V}}_{\mathrm{ab}}$, can be obtained by Kirchhoff's voltage law (KVL- the sum of all phasor voltage drops around any path in a circuit equals zero):

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn} = V\angle 0° - V\angle -120° = V[1-(-0.5 - j0.866)] = \sqrt{3}V\angle 30° = \sqrt{3}\overline{V}_{an}\angle 30°.$$

Similarly, the line-to-line voltages $\overline{V}_{bc}$ and $\overline{V}_{ca}$ can be obtained

$$\overline{V}_{bc} = \sqrt{3}V\angle -90° = \sqrt{3}\overline{V}_{bn}\angle 30°,$$

$$\overline{V}_{ca} = \sqrt{3}V\angle 150° = \sqrt{3}\overline{V}_{cn}\angle 30°.$$

Therefore, for a wye connected balanced three-phase voltage source, the line-to-line voltages, or line voltages, are $\sqrt{3}$ times the phase voltages in magnitude and the line voltages are 30° ahead of their coresponding phase voltages. Figure 1-11 shows a graphical presentation of these relationships.

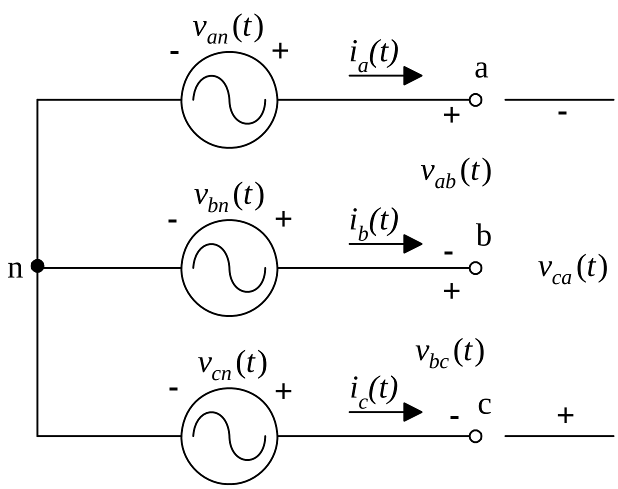

Fig. 1-10. Three-phase voltage source with wye configuration.

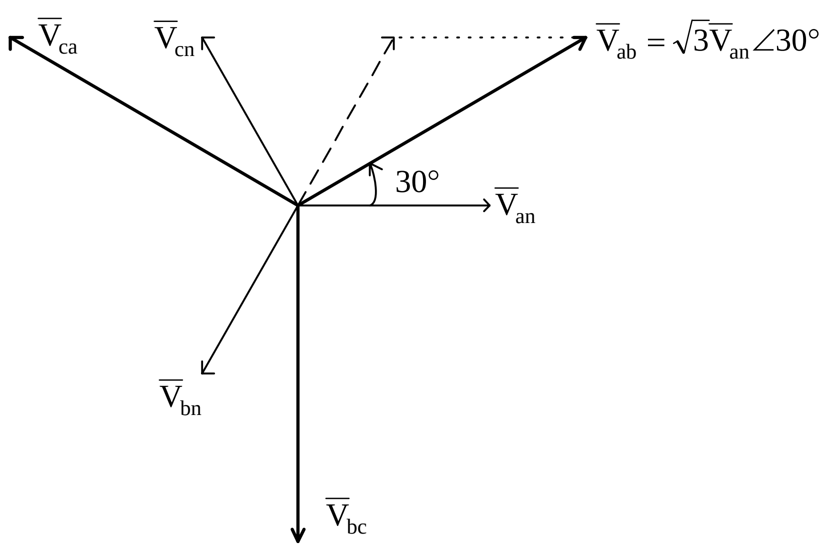

Fig. 1-11. The relationships of phase voltages and line-to-line voltages in a three-phase system with wye configuration.

As one can see from the Figure 1-10, the phase currents are the same as the line currents for a wye connected voltage source.

The other configuration of the three-phase voltage source is to connect them in a delta configuration, as shown in Figure 1-12. As one can see, the phase voltages are the same as their corresponding line voltages in such a delta configuration. However, the line currents and phase currents are different, and their relationships can be derived similar to the voltage relationships in a wye configuration, namely, at node a by Kichhoff's current law (KCL- the sum of all phasor currents into any nodes in a circuit equals zero)

$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = I\angle 0° - I\angle 120° = I[1-(-0.5 + j0.866)] = \sqrt{3}I\angle -30° = \sqrt{3}\bar{I}_{ab}\angle -30°.$$

Similarly, at nodes b and c, the other two line currents can be obtained as follows

$$\bar{I}_b = \sqrt{3}I\angle -150° = \sqrt{3}\bar{I}_{bc}\angle -30°,$$

$$\bar{I}_c = \sqrt{3}I\angle 90° = \sqrt{3}\bar{I}_{ca}\angle -30°.$$

Therefore, for a delta connected balance three-phase voltage source, the line currents are $\sqrt{3}$ times the phase currents in magnitude and the line currents are 30° behind of their coresponding phase currents. Figure 1-13 shows a graphical presentation of these relationships.

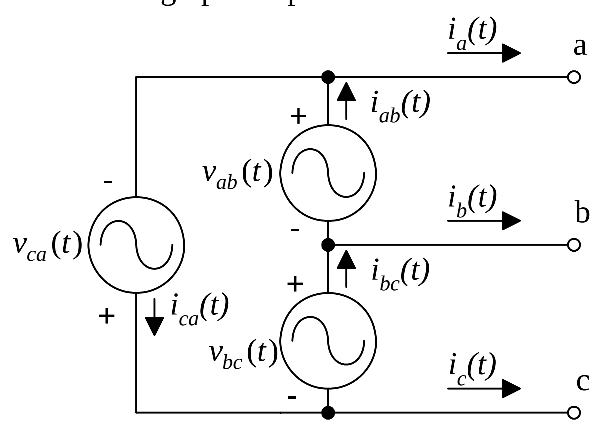

Fig. 1-12. Three-phase voltage source with delta configuration.

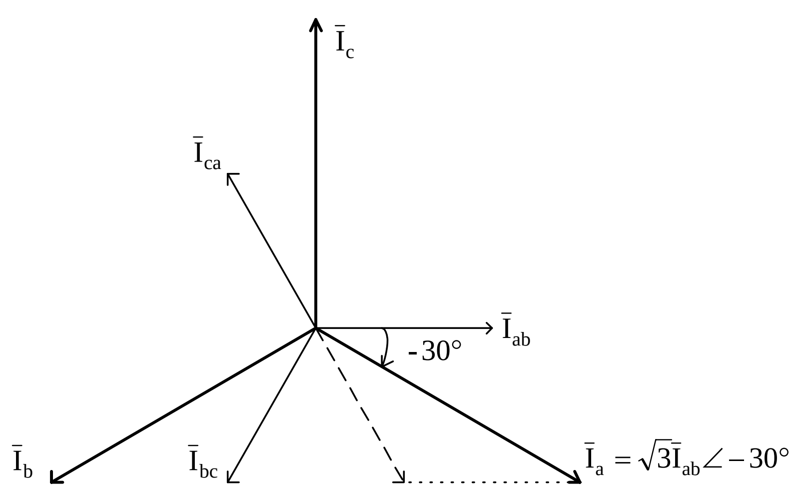

Fig. 1-13. The relationships of phase currents and line currents in a three-phase system with delta configuration.

A "balanced" three-phase system has their phase/line voltages mutually shifted by 120°, and their phase/line currents has the same property. Since it is balanced, the power for each phase is the same, for instance, the per phase complex power

$$\overline{S}_{an} = \overline{S}_{bn} = \overline{S}_{cn} = \overline{S}_{1\phi},$$

or

$$\overline{S}_{ab} = \overline{S}_{bc} = \overline{S}_{ca} = \overline{S}_{1\phi}.$$

Therefore, the three-phase complex power equals three times of its per phase value, namely,

$$\overline{S}_{3\phi} = 3\overline{S}_{1\phi} = 3V_\phi I_\phi \angle\phi,$$

where V_ϕ and I_ϕ are the phase voltage and the phase current, respectively. As discussed earlier, for a wye connected voltage source, the magnitude of the line voltage is $\sqrt{3}$ times of the phase voltage, while the line and phase current are the same. Therefore, the three-phase apparent power is equal to

$$S_{3\phi} = 3V_\phi I_\phi = 3\frac{V_L}{\sqrt{3}} I_L = \sqrt{3} V_L I_L,$$

where V_L and I_L are the rms values of the line voltage and line current, respectively. This equation can be applied to a delta connected source as well.

Then, the three-phase active power, reactive power and power factor can be obtained as follows

$$P_{3\phi} = 3V_\phi I_\phi \cos\phi = \sqrt{3} V_L I_L \cos\phi,$$

$$Q_{3\phi} = 3V_\phi I_\phi \sin\phi = \sqrt{3} V_L I_L \sin\phi,$$

and

$$pf_{3\phi} = \frac{P_{3\phi}}{S_{3\phi}} = \frac{P_{1\phi}}{S_{1\phi}} = \cos\phi.$$

Example 1-4: A balanced three-phase load of 50 kVA, pf = 0.85 lagging is supplied from a balanced three-phase wye connected voltage source of positive sequence such that V_L = 4157 volts. Calculate:

(1) I_L, V_ϕ and I_ϕ.

(2) $S_{3\phi}$, $P_{3\phi}$ and $Q_{3\phi}$.

Solution:

(1) $V_\phi = \frac{V_L}{\sqrt{3}} = \frac{4157}{\sqrt{3}} = 2400$ volts

$$I_\phi = I_L = \frac{S_{3\phi}}{\sqrt{3}V_L} = \frac{50\times 10^3}{\sqrt{3}\times 4157} = 6.94 \text{ amps}$$

(2) $\overline{S}_{3\phi} = 50\angle\cos^{-1}(0.85) = 50\angle 31.8^\circ \text{ kVA}$

$$S_{3\phi} = 50 \text{ kVA}$$

$$P_{3\phi} = 50\times 0.85 = 42.5 \text{ kW}$$

$$Q_{3\phi} = 50\sin[\cos^{-1}(0.85)] = 50\sin 31.8^\circ = 26.34 \text{ kvar}$$ ♦

Per Unit Values

In power system calculations, it is very often to normalize actual values, such as voltages and currents, to per unit values. It is very convenient if many transformers and voltage levels are involves. The per unit value is defined as the ratio of the actual value to the selected base value, namely,

$$X_{pu} = \frac{X_{Actual}}{X_{Base}},$$

where X can be the power, voltage, current and/or impedance. However, usually the base voltage and base power (VA) are given quantities while the base current and base impedance are to be determined accordingly.

Example 1-5: A motor is rated 4.16 kV and can be operated +/- 10% of its rated voltage (1.1~0.9 pu). What is the range of the operational voltage in kV?
Solution:

The high limit voltage is 1.1 pu and its actual voltage can be calculated as
$V_{Hi} = 1.1\times 4.16 = 4.576 \quad \text{kV}$.
Similarly, the low limit voltage is
$V_{Lo} = 0.9\times 4.16 = 3.744 \quad \text{kV}$.
The motor can be operated in the range of 4.576~3.744 kV. ♦

For single-phase systems:

(1) Select $S_{Base,1\phi} = S_{1\phi}$, and $V_{Base,1\phi} = V_\phi$.

(2) Then, $I_{Base,1\phi} = \frac{S_{Base,1\phi}}{V_{Base,1\phi}} = \frac{S_{1\phi}}{V_\phi}$, and $Z_{Base,1\phi} = \frac{V_{Base,1\phi}}{I_{Base,1\phi}} = \frac{V^2_{Base,1\phi}}{S_{Base,1\phi}}$.

(3) Therefore, $\overline{S}_{pu} = \dfrac{S_{Actual}\angle\phi}{S_{Base,1\phi}} = \dfrac{P_{Actual}}{S_{Base,1\phi}} + j\dfrac{Q_{Actual}}{S_{Base,1\phi}} = P_{pu} + jQ_{pu}$,

$$\overline{V}_{pu} = \frac{\overline{V}_{Actual}}{V_{Base,1\phi}},$$

$$\overline{I}_{pu} = \frac{\overline{I}_{Actual}}{I_{Base,1\phi}}, \text{ and}$$

$$\overline{Z}_{pu} = \frac{\overline{Z}_{Actual}}{Z_{Base,1\phi}}.$$

Once the per unit values are calculated, the actual values can be obtained by multiplying the per unit values with their corresponding base values.

Example 1-6: A single-phase system, as shown in Figure 1-6, is given in actual values. Find:

(1) I_{Base} and Z_{Base} with selecting S_{Base} = 7.2 kVA and V_{Base} = 120 volts.

(2) $\overline{V}_{pu}$, $\overline{I}_{pu}$, $\overline{Z}_{pu}$, R_{pu}, and X_{pu}.

(3) The current in amperes.

Solution:

(1) $I_{Base} = \dfrac{7200}{120} = 60$ amps

$$Z_{Base} = \frac{120^2}{7200} = 2\ \Omega$$

(2) Set the voltage as the reference, 0°. Then

$$\overline{V}_{pu} = \frac{120\angle 0°}{120} = 1\angle 0° \text{ pu}$$

$$\overline{Z}_{pu} = \frac{10 + j10}{2} = 5 + j5 = R_{pu} + jX_{pu} \text{ pu}$$

$$R_{pu} = 5 \text{ pu}$$

$$X_{pu} = 5 \text{ pu}$$

$$\overline{I}_{pu} = \frac{1\angle 0°}{5\sqrt{2}\angle 45°} = 0.14142\angle -45° \text{ pu}$$

(3) $\overline{I} = I_{Base} \times \overline{I}_{pu} = 60 \times (0.14142\angle -45°) = 8.485\angle -45°$ amps

which is the same as in the Example 1-3. ♦

Similarly, for three-phase systems:

(1) Select $S_{Base,3\phi} = S_{3\phi} = 3S_{1\phi}$, and $V_{Base,3\phi} = V_{Base,L} = V_L = \sqrt{3}V_{\phi} = \sqrt{3}V_{Base,1\phi}$.

(2) Then, $I_{Base,3\phi} = \dfrac{S_{Base,1\phi}}{V_{Base,1\phi}} = \dfrac{\frac{1}{3}S_{Base,3\phi}}{\frac{1}{\sqrt{3}}V_{Base,3\phi}} = \dfrac{S_{Base,3\phi}}{\sqrt{3}V_{Base,3\phi}}$, and

$$Z_{Base,3\phi} = \frac{V^2_{Base,1\phi}}{S_{Base,1\phi}} = \frac{\frac{1}{3}V^2_{Base,3\phi}}{\frac{1}{3}S_{Base,3\phi}} = \frac{V^2_{Base,3\phi}}{S_{Base,3\phi}}.$$

(3) Therefore, $\overline{S}_{pu} = \dfrac{S_{Actual}\angle\phi}{S_{Base,3\phi}}$,

$$\overline{V}_{pu} = \frac{\overline{V}_{Actual}}{V_{Base,3\phi}},$$

$$\overline{I}_{pu} = \frac{\overline{I}_{Actual}}{I_{Base,3\phi}}, \text{ and}$$

$$\overline{Z}_{pu} = \frac{\overline{Z}_{Actual}}{Z_{Base,3\phi}}.$$

From time to time, it is necessary to change the per unit value of the impedance from one base to another. The following equation can be used for this conversion:

$$\overline{Z}_{pu(new)} = \overline{Z}_{pu(old)}\left(\frac{V_{Base(old)}}{V_{Base(new)}}\right)^2\left(\frac{S_{Base(new)}}{S_{Base(old)}}\right).$$

Proof: (Hint: when changing the impedance per unit value from one base to another, its actual value does not change)

$$\overline{Z}_{pu(new)} = \frac{\overline{Z}_{Actual}}{Z_{Base(new)}} = \frac{\overline{Z}_{pu(old)}\left(\frac{V^2_{Base(old)}}{S_{Base(old)}}\right)}{\frac{V^2_{Base(new)}}{S_{Base(new)}}} = \overline{Z}_{pu(old)}\left(\frac{V_{Base(old)}}{V_{Base(new)}}\right)^2\left(\frac{S_{Base(new)}}{S_{Base(old)}}\right)$$

Example 1-7: An impedance of $\overline{Z}_{pu} = 0.05 + j0.5$ pu on $S_{Base,3\phi} = 200\text{MVA}$ and $V_{Base,L} = 138\text{kV}$. Calculate the new $\overline{Z}_{pu}$ if new base values for $S_{Base,3\phi}$ and $V_{Base,L}$ are given as

(a) 100MVA, 138 kV
(b) 200MVA, 132 kV

(c) 100MVA, 132 kV, respectively.

Solution:

(a) $\overline{Z}_{pu} = (0.05 + j0.5)\left(\frac{100}{200}\right) = 0.025 + j0.25 \text{ pu}$

(b) $\overline{Z}_{pu} = (0.05 + j0.5)\left(\frac{138}{132}\right)^2 = 0.05465 + j0.5465 \text{ pu}$

(c) $\overline{Z}_{pu} = (0.05 + j0.5)\left(\frac{138}{132}\right)^2\left(\frac{100}{200}\right) = 0.02732 + j0.2732 \text{ pu}$ ♦

Example 1-8: A three-phase equipment is rated at 800 kVA, 12 kV and has an impedance of 0.005+j0.1 pu referred to its ratings. Calculate the impedance in ohms.

Solution:

$$\overline{Z}_{pu} = \frac{\overline{Z}_{Actual}}{Z_{Base,3\phi}} \text{ pu.}$$

Therefore,

$$\overline{Z}_{Actual} = \overline{Z}_{pu} \times Z_{Base,3\phi} = (0.005 + j0.1) \times \frac{(12 \times 10^3)^2}{800 \times 10^3} = (0.005 + j0.1) \times 180 = 0.9 + j18\ \Omega$$

♦

Appendix 1A
Complex Numbers and Polar Coordinates

A complex number can be expressed in terms of its real component and its imaginary component in the complex plane. For instance, 3 + j 4 can be placed on the complex plane as shown in Fig. A1. For the same complex number, it can be expressed in terms of its modulus (magnitude) and angle, namely,

$$3 + j4 = \left(\sqrt{3^2 + 4^2}\right)\angle \tan^{-1}\left(\frac{4}{3}\right) = 5\angle 53.13° \,.$$

The magnitude is the distance of the complex number to origin, and the angle (or argument) is the counterclockwise angle from the real-axis (0°) of the complex number.

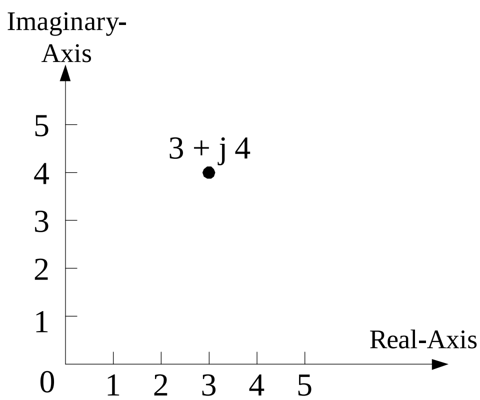

Fig A1. 3 + j 4 in complex plane.

The same complex number is shown in polar representation in Fig. A2.

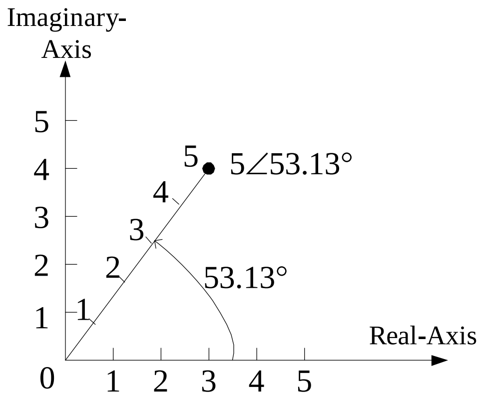

Fig. A2. Polar representation of 3 + j 4 in complex plane.

Example A1: Convert 1.15 + j 0.9 into its Polar coordinate.
Solution:

$$1.15 + j0.9 = \left(\sqrt{1.15^2 + 0.9^2}\right)\angle \tan^{-1}\frac{0.9}{1.15} = 1.4603\angle 38.05° \qquad \blacklozenge$$

Example A2: Convert $10\angle 30°$ into its trigonometric form.
Solution:

$$10\angle 30° = 10(\cos 30° + j\sin 30°) = 10(0.866 + j0.5) = 8.66 + j5$$ ♦

Example A3: Adding $10\angle 30°$ and 1.15 + j 0.9 and express the sum in trigonometric form and polar form.
Solution:

The sum in trigonometric form:

$$10\angle 30° + 1.15 + j0.9 = 8.66 + j5 + 1.15 + j0.9 = 9.81 + j5.9$$

The sum in Polar form:

$$10\angle 30° + 1.15 + j0.9 = 9.81 + j5.9 = \left(\sqrt{9.81^2 + 5.9^2}\right)\angle \tan^{-1}\frac{5.9}{9.81} = 11.4475\angle 31.02°$$

♦