PRACTICAL CIRCUIT ANALYSIS

Richard W. Fruge, P.E. (Inactive)

I. Component Circuit Parameters

The circuit shown in Figure 1 will be used throughout this module to demonstrate the methods available for circuit analysis. This demonstration will include use of a schematic diagram and the development of circuit parameters using equipment data obtained from specification sheets. This static analysis is valid only at the instant voltage is applied to the closing coil of the motor starter.

Equipment specifications are rarely provided in a format that can be used without manipulation. The data required to perform time domain analysis are not readily available, and in most cases, evaluation at time = 0 or time = steady state can be evaluated "statically" using published equipment specifications. Methods, using basic engineering principles, are established to convert the data provided into a format that supports solution of the circuit being studied.

The circuit in Figure 1 is taken from a control wiring diagram having multiple circuit paths available for operating the opening and closing coils of the motor starter. This normally includes local operation, remote manual operation, and remote automatic operation. Only analysis of the "worst case" path is performed because it "envelopes" all the other cases. For the purposes of this analysis, actuation of the starter coil is analyzed by taking the circuit path with the largest conductor impedance.

Control Power Transformer parameters typically provided are as follows:

VA	150
Primary Voltage	480
Secondary Voltage	120
Open Circuit Voltage	126
% R	3.8
% X	2.2

To convert the % R and % X values to ohm values, the relationship between apparent power, voltage, current, and impedance is used. Recall that the transformer rating is given by:

$$|S| = |V \times I| = \left|\frac{V^2}{Z}\right| \quad \text{VA}$$

Therefore:

$$|Z| = \frac{|V|^2}{|S|} = \frac{120^2}{150} = 96 \quad \Omega$$

$$R = 96 \times \frac{3.8}{100} = 3.648 \quad \Omega$$

$$X = 96 \times \frac{2.2}{100} = 2.112 \quad \Omega$$

The result above is now applied to the schematic diagram in Figure 1 as follows:

Relay (74) parameters typically provided are as follows:

VA	15 (Holding)	120 (Inrush)
Power Factor	0.45 (Holding)	0.6 (Inrush)
Voltage	120	

For analyzing the circuit shown in Figure 1, the 74 device is energized when 480 V is present at the MCC and the "holding" impedance representation is required to analyze this configuration. From the data provided:

$$|S| = |V \times I| = \left|\frac{V^2}{Z}\right|$$

Therefore:

$$|Z| = \frac{|V|^2}{|S|} = 960 \quad \Omega$$

Also,

$$R = \mathrm{Re}(Z)$$

Using the above

$$R = \frac{V^2 \times PF}{|S|} = \frac{120^2 \times 0.45}{15} = 432 \quad \Omega$$

$$X = \sqrt{Z^2 - R^2} = 857.307 \quad \Omega$$

The result above is now applied to the schematic diagram in Figure 1 as follows:

$$Z_{74} = 432 + j857.307 \quad \Omega$$

Relay (3X) parameters typically provided are as follows:

VA	80 (Inrush)
Power Factor	0.4 (Inrush)
Voltage	120

For analyzing the circuit shown in Figure 1, the 3X device is energized at the same instant the starter coil is energized and the "inrush" impedance representation is required to analyze this configuration. From the data provided:

$$|S| = |V \times I| = \left|\frac{V^2}{Z}\right|$$

Therefore:

$$|Z| = \frac{|V|^2}{|S|} = 180 \quad \Omega$$

Also,

$$R = \mathrm{Re}(Z)$$

Using the above

$$R = \frac{V^2 \times PF}{|S|} = \frac{120^2 \times 0.4}{80} = 72 \quad \Omega$$

$$X = \sqrt{Z^2 - R^2} = 164.973 \quad \Omega$$

The result above is now applied to the schematic diagram in Figure 1 as follows:

$$Z_{3X} = 72 + j164.973 \quad \Omega$$

Light Circuit Parameters

The panel light is considered because it represents a circuit burden that tends to lower the voltage at downstream devices. Due to large impedance of the light, the cable impedance can usually be omitted.

40 mA @ 28 V with a 3000 Ω Resistor

$$R_{light} = 3000 + \frac{28}{0.040} = 3700\Omega$$

Motor Starter parameters typically provided are as follows:

VA	180 (Inrush)
I	1.5 (Inrush)
Power Factor	0.6 (Inrush)
Voltage	120

For analyzing the circuit shown in Figure 1, the 42 device is energized when 120 V is present at the coil and the "inrush" impedance representation is required to analyze this configuration. From the data provided:

$$|S| = |V \times I| = \left|\frac{V^2}{Z}\right|$$

Therefore:

$$|Z| = \frac{|V|^2}{|S|} = 80 \quad \Omega$$

Also,

$$R = \mathrm{Re}(Z)$$

Using the above,

$$R = \frac{V^2 \times PF}{|S|} = \frac{120^2 \times 0.6}{180} = 48 \quad \Omega$$

$$X = \sqrt{Z^2 - R^2} = 64 \quad \Omega$$

The result above is now applied to the schematic diagram in Figure 1 as follows:

$$Z_{42} = 48 + j64 \quad \Omega$$

Circuit Conductor Impedance

The circuit impedance represented in Figure 1 as Z1, Z2, and Z3 is derived from the following:

Vendor supplied resistance values for control cable are based on 25°C, but our study requires impedance values based on 40°C operation. The cable vendor provides the following information for impedance at other temperatures:

$$R_{40} = R_1 \frac{(234.5 + T_2)}{(234.5 + T_1)} = R_1 \frac{(234.5 + 40.0)}{(234.5 + 25)} = 1.058 R_1$$

Impedance is then converted to 40°C values for the following:

Cable Type	Ω/1000 ft @ 25°C	Ω/1000 ft @ 40°C
A	7.05	7.46
B	4.44	4.70
C	2.73	2.89
D	1.83	1.93
E	0.91	0.96
F	1.72	1.82
G	1.08	1.14

NOTE: In the absence of specific cable data, the NEC provides typical impedance values for various cable types, as well as methodology for evaluating temperature effects.

Z1 is 400 feet of "B" cable , Z2 is 1590 feet of "B" cable, and Z3 is 150 feet of "B" cable. We then convert this data into ohmic values (neglecting reactance) as follows:

$$Z = \frac{l \times R}{1000}\ \Omega$$

Therefore:

$Z_1 = 1.879\ \Omega$; $Z_2 = 7.468\ \Omega$; $Z_3 = 0.704\ \Omega$

Fuse Impedance

For the purposes of this analysis, the impedance of the control circuit fuse, Z_f is ignored. It should be noted, however, that the fuse represents finite impedance and will tend to reduce the magnitude of the voltage applied to downstream components. For marginal cases, it may be prudent to include this impedance in the calculation.

II. Circuit Reduction Methodology

This section demonstrates analysis that cannot be readily automated, and is typical of many early calculations. The purpose of this analysis is to show that this technique, although valid, is tedious even when implemented using common math software packages. Validation must include verification of the correctness of all intermediate steps used to calculate the equivalent impedance, as well as the intermediate steps used to calculate the starter current. It is apparent that a minor math error would be carried through the remainder of the calculation, making detection difficult.

The circuit in Figure 1 is simple and circuit reduction techniques can be used for analysis. Note that kEoc is used for the source voltage. "k" is used to denote the fraction of bus voltage available at the control power transformer primary (i.e. MCC bus voltage). This representation provides CPT data as well as initial circuit conditions. Later, when calculation automation issues are addressed, analysis of the circuit at various MCC bus levels is achieved by changing the value of "k". k = 1.0 for the circuit shown in Figure 1.

The equivalent impedance is calculated by combining the series and parallel elements of the circuit. The circuit is then expanded in the reverse order and current division is used to calculate the inrush current flowing through the starter coil.

Now that the current and impedance are known quantities, the voltage is easily calculated.

1. Circuit parameters:

$Zf=0$; $Z1=1.879$; $Z_2=7.468$; $Z_3=0.704$; $Z_{lgt}=3700$; $Z_{74}=432+j857.307$

$Z_{42}=48+j64$; $Z_{3X}=72+j164.973$; $Z_{cpt}=3.648+j2.112$

$E_{oc}=126$; $k=1$

2. Calculation of the Equivalent Impedance:

$$Z_{a1}=Z_2+Z_{42} \qquad Z_{a2}=Z_3+Z_{3X}$$

$$Z_a=\frac{Z_{a1}\times Z_{a2}}{Z_{a1}+Z_{a2}}$$

$$Z_b=Z_a+Z_1$$

$$Z_{c1}=\frac{Z_{\lg t}\times Z_b}{Z_{\lg t}+Z_b} \qquad Z_c=\frac{Z_{c1}\times Z_{74}}{Z_{c1}+Z_{74}}$$

$$Z_{eq}=Z_{cpt}+Z_f+Z_c$$

$$Z_{eq}=37.016+j46.330\ \Omega$$

3. Calculate I to Starter

$$i_1=\frac{kE_{oc}}{Z_{eq}}=1.326-j1.660 \qquad i_2=\frac{Z_{74}}{Z_{c1}+Z_{74}}\times i_1=1.268-j1.552$$

$$i_3=\frac{Z_{\lg t}}{Z_b+Z_{\lg t}}\times i_2=1.236-j1.553 \qquad i_4=\frac{Z_{a2}}{Z_{a1}+Z_{a2}}\times i_3=0.947-j0.981$$

Calculate Voltage Magnitude at the Starter

$$V_{42}=i_4\times Z_{42}=108.261+j13.501$$

$$|V_{42}|=109.1$$

It is apparent at this point that if a circuit component is changed, the calculation in section I would require revision, then the circuit analysis in section II (this section) would be performed using the new component parameters. Evaluation of component changes will be discussed further in section IV.

III. Analysis Using Cramer's Rule

The control circuit in Figure 1 is used to write the mesh equations. Current i1 is defined as the current flowing through the "loop" formed by Zcpt, Zf , and Z74.(Nodes 1, 2, 3, and 0). Current i2 is defined as the current flowing through the "loop" formed by Zlgt and Z74 (Nodes 3 and 0). Current i3 is defined as the current flowing through the "loop" formed by Z1, Z2, Z42, and Zlgt (Nodes 3, 4, 5, and 0). Current i4 is defined as the current flowing through the "loop" formed by Z2, Z3, Z3X, and Z42 (Nodes 4, 5, 6, and 0). The circuit equations are written as follows:

$$kE_{oc} = \left(Z_{cpt} + Z_f + Z_{74}\right) i_1 - \left(Z_{74}\right) i_2$$

$$0 = \left(-Z_{74}\right) i_1 + \left(Z_{74} + Z_{lgt}\right) i_2 - \left(Z_{lgt}\right) i_3$$

$$0 = -\left(Z_{lgt}\right) i_2 + \left(Z_1 + Z_2 + Z_{42} + Z_{lgt}\right) i_3 - \left(Z_2 + Z_{42}\right) i_4$$

$$0 = -\left(Z_2 + Z_{42}\right) i_3 + \left(Z_2 + Z_3 + Z_{3X} + Z_{42}\right) i_4$$

The circuit equations above can be easily developed by inspection, if the engineer is familiar with circuit analysis. However, when circuit analysis is performed infrequently, this can present a challenge and require time for the engineer to get reacquainted with the analysis method.

The matrix notation for the equations above is as follows:

$$[V] = [i][Z]$$

and substituting circuit specific components,

$$\begin{bmatrix} kE_{oc} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \begin{bmatrix} Z_{cpt} + Z_f + Z_{74} & -Z_{74} & 0 & 0 \\ -Z_{74} & Z_{74} + Z_{lgt} & -Z_{lgt} & 0 \\ 0 & -Z_{lgt} & Z_1 + Z_2 + Z_{42} + Z_{lgt} & -\left(Z_2 + Z_{42}\right) \\ 0 & 0 & -\left(Z_2 + Z_{42}\right) & \left(Z_2 + Z_3 + Z_{3X} + Z_{42}\right) \end{bmatrix}$$

Circuit solution is now possible using Cramer's Rule or Gauss-Siedel iteration. Since the number of arithmetic steps required to solve the four simultaneous equations using determinants is not excessive, Cramer's Rule will be used to provide a solution for i3 and i4 , which will be used to determine the value of V_{42}. A review of Cramer's Rule can be found at http://www.sosmath.com/matrix/determ2/determ2.html.

1. Solution Using Cramer's Rule requires calculation of loop currents i_3 and i_4, since the current flowing through the coil is the vector sum of these currents.

$$i_3 = \frac{\begin{vmatrix} Z_{cpt} + Z_f + Z_{74} & -Z_{74} & kE_{oc} & 0 \\ -Z_{74} & Z_{74} + Z_{lgt} & 0 & 0 \\ 0 & -Z_{lgt} & 0 & -\left(Z_2 + Z_{42}\right) \\ 0 & 0 & 0 & \left(Z_2 + Z_3 + Z_{3X} + Z_{42}\right) \end{vmatrix}}{\begin{vmatrix} Z_{cpt} + Z_f + Z_{74} & -Z_{74} & 0 & 0 \\ -Z_{74} & Z_{74} + Z_{lgt} & -Z_{lgt} & 0 \\ 0 & -Z_{lgt} & Z_1 + Z_2 + Z_{42} + Z_{lgt} & -\left(Z_2 + Z_{42}\right) \\ 0 & 0 & -\left(Z_2 + Z_{42}\right) & \left(Z_2 + Z_3 + Z_{3X} + Z_{42}\right) \end{vmatrix}}$$

$$i_4 = \frac{\begin{vmatrix} Z_{cpt}+Z_f+Z_{74} & -Z_{74} & 0 & kE_{oc} \\ -Z_{74} & Z_{74}+Z_{\lg t} & -Z_{\lg t} & 0 \\ 0 & -Z_{\lg t} & Z_1+Z_2+Z_{42}+Z_{\lg t} & 0 \\ 0 & 0 & -(Z_2+Z_{42}) & 0 \end{vmatrix}}{\begin{vmatrix} Z_{cpt}+Z_f+Z_{74} & -Z_{74} & 0 & 0 \\ -Z_{74} & Z_{74}+Z_{\lg t} & -Z_{\lg t} & 0 \\ 0 & -Z_{\lg t} & Z_1+Z_2+Z_{42}+Z_{\lg t} & -(Z_2+Z_{42}) \\ 0 & 0 & -(Z_2+Z_{42}) & (Z_2+Z_3+Z_{3X}+Z_{42}) \end{vmatrix}}$$

$$V_{42} = (i_3 - i_4)Z_{42}$$

$$|V_{42}| = 109.1$$

Although the solution is identical, it is clear that use of matrices and Cramer's Rule is more straightforward and can be easily solved using commercial off-the-shelf math programs. However, like the reduction techniques above, automating this method would not be a trivial task.

It is again apparent that if a circuit component is changed, the calculation in section I would require revision, then the circuit analysis in section III (this section) would be performed using the new component parameters. When using commercial math software, this would entail changing the component parameters only. The equations would then be automatically recalculated and the solution available. Evaluation of component changes will be discussed further in section IV.

It is interesting to note at this point that if commercial math software is used for calculations, the matrix "dot product" could be used to further simplify matrix solutions. Unlike Cramer's Rule, the matrix "dot product" solution calculates all loop currents in one mathematical operation. $[i] = [Y] \bullet [V] \ where \ [Y] = [Z]^{-1}$ solves for i_1 through i_4 and eliminates the determinate solutions required by Cramer's Rule.

The next section describes the automation of circuit solutions using Gauss-Seidel methods.

IV. Automating Circuit Solutions

Since solution using Gauss-Seidel and Cramer's Rule are similar (i.e. mesh equations are developed), this section will introduce tools that can be used to solve the set of simultaneous equations developed in the section above. Gauss-Seidel is an "iterative" technique for the solution of simultaneous equations that can be implemented easily using commercial software products.

Figure 2 shows the output of a circuit analysis program developed using Microsoft Access 97. The program implements Gauss-Seidel solution of N simultaneous equations. Use of a database eliminates the need for specially formatted data files and provides an intuitive user interface (input forms). Additionally, use of a database allows for storage of component data for reuse in many circuit case studies. Stand-alone forms are designed to perform and store the results of specialized, repetitive calculations (transformer impedance, cable impedance calculator, and relay impedance calculator) and insert them into the current study case. All the attributes and properties of a relational database are used to minimize required input.

The program dynamically calculates the number of equations, N, and assembles the matrices and provides the solution. The flexible reporting capabilities provide a means for automating the calculation process. Note that the voltage drop across all circuit components can be calculated. Development of programs of this type eliminate the computational aspects of circuit analysis and requires only circuit input in the "From – To" format. This feature is especially efficient when this type of analysis is performed on an infrequent basis by eliminating the learning curve. An investment of 200 to 300 man hours would be required to develop a similar program, but a facility requiring the analysis of several MCC's would realize significant time savings through calculation automation. The possibilities for automation are endless, and customization to meet specific requirements is easily achieved by engineers with minimal programming skills.

Program development efforts should include consideration of the following:

- Verification/Validation requirements of corporate software QA programs
- Education/experience level of end-users
- Development time Vs. projected task time savings
- Flexibility, can perform multiple tasks/calculations
- Simplicity, based on "first principles"
- Expertise and resources required for proper implementation
- Corporate Return on Investment (ROI) standards

Students using this module for PDH credit may request a free copy of a "tool box" containing the circuit analysis tool described above. This will enable them to explore the benefits that could be realized by automating repetitive tasks, as well as provide them with a sense of what is involved in the application of an automated approach to calculation systems. Email the author to request your free copy of the tool at rwfruge@yahoo.com.

The flexibility of this automated calculation tool will now be demonstrated by analyzing several "what if" examples using the circuit analysis tool described above.

1. What if you double the X/R ratio of the motor starter ($Z_{42} = R + j2X$)? Compare the results with Figure 2.

   You would copy the Figure 1 circuit data and paste to a new record (Example 1). Change the X value for Z_{42} then press CALCULATE. Use HEADERS to prepare the desired report header, then press PRINT RESULTS. When compared to the results in Figure 2, Z_{eq} is increased and the voltage at Z_{42} has increased by approximately 7.5 V.

2. What if you double the X/R ratio of the motor starter ($Z_{42} = R/2 + jX$)? Compare the results with Figure 2.

   You would copy the Figure 1 circuit data and paste to a new record (Example 2). Change the R value for Z_{42} then press CALCULATE. Use HEADERS to prepare the desired report header, then press PRINT RESULTS. When compared to the results in Figure 2, Z_{eq} is decreased and the voltage at Z_{42} has increased by approximately 1.5 V.

3. The manufacturer of the motor starter specifies a minimum pickup voltage of 80% (0.8 x 120V = 96V). What if the MCC bus voltage is degraded to 86.5% nominal (k = 0.865)? Would the voltage specification for minimum pickup be satisfied?

   You would copy the Figure 1 circuit data and paste to a new record (Example 3). Change the k value then press CALCULATE. Use HEADERS to prepare the desired report header, then press PRINT RESULTS. When compared to the results in Figure 2, the voltage at Z_{42} has decreased to 94.37 V. The minimum pickup specification is not met, and the starter will not "pick up".

Since it is based on fundamental analysis methods, this calculation tool can be used to evaluate any circuit. For example, if you are required to determine the impedance of parallel transformers, a transformer model can be used as the basis for circuit connections in the "From – To" format. The solution includes calculation of Z_{eq}, which is the desired value.

Flexibility is probably the most important consideration when designing tools to automate a calculation process. It is interesting to note that the flexibility of a "calculation tool" is generally inversely proportional to the complexity. Tools developed using "first principles" are easier to expand and inherently more flexible than those based on advanced concepts.

This should not deter the engineer from developing a task specific tool. Flexibility can evolve over time as additional uses of the tool are discovered, sometimes requiring minor changes to accommodate the new functions.

V. Summary

Methods are described to analyze control circuits for proper operation in all component configurations. Circuit reduction and Cramer's Rule solution techniques are presented. Methods for converting manufacturer's component parameters to values required for circuit analysis using basic Engineering fundamentals are also presented. An automated circuit analysis tool using Gauss-Seidel methods is provided to demonstrate calculation automation possibilities.

Design considerations are provided for automated calculation program efforts. Flexibility and simplicity are identified as important characteristics for engineering "tools".

FIGURE 1 - Typical 480 Volt Motor Starter Control

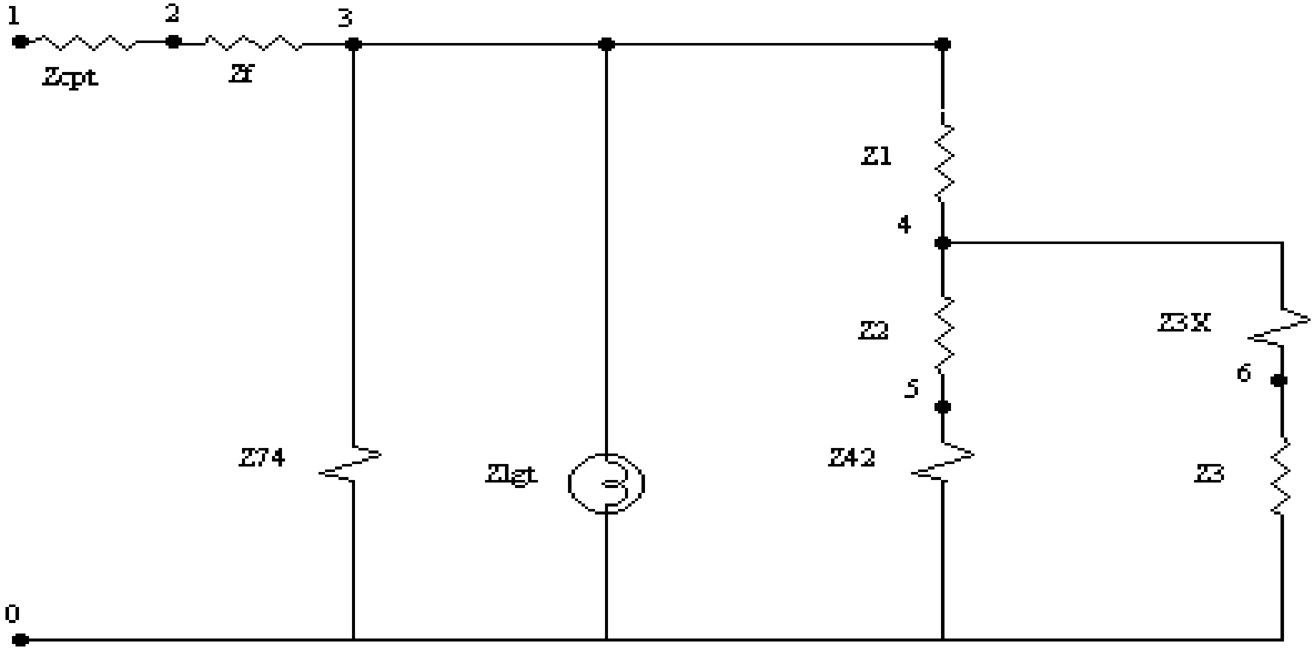

FIGURE 2 – Sample Output

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Circuit Analysis Program                                          Figure 2
Circuit Solution #1                                               Page 1 of  1

 Circuit: Figure 1                                   Schematic
        Vs  126.000 + j0.000   k: 1.000              Zeq   37.016 + j46.330
Circuit Description:
        This circuit is for the demonstration of automated Gauss-Seidel method for calculating the
        voltage drop in a typical control circuit of a 480V MCC.
Input Data
Device                From     To       Resistance Reactance
CPT-150               1        2        3.648      2.112
Zf                    2        3        0.000      0.000
Zlgt                  3        0        3700.000   0.000
Z74 (Holding)         3        0        432.000    857.307
Z1                    3        4        1.879      0.000
Z2                    4        5        7.468      0.000
Z3X (Inrush)          4        6        72.000     164.973
Z42 (Inrush)          5        0        48.000      64.000
Z3                    6        0        0.704      0.000
 Results
      Device                            Voltage        Device                          Voltage
Zlgt                                    117.700        Z74 (Holding)                   117.700
Z42 (Inrush)                            109.100        Z3X (Inrush)                    115.809
Z3                                      0.450          Z2                              10.180
Z1                                      3.730          CPT-150                         8.960
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