PDHonline Course M125 (3 PDH)

Basic Pump Parameters and the Affinity Laws

Instructor: Randall W. Whitesides, P.E.

2020

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

An Approved Continuing Education Provider

Basic Pump Parameters and the Affinity Laws

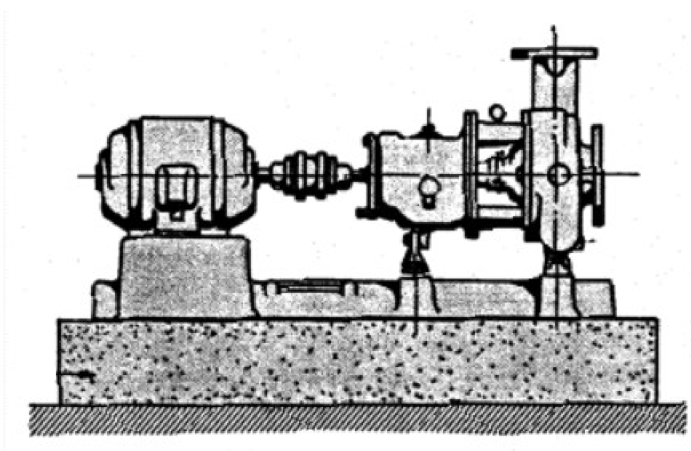
Randall W. Whitesides P.E.

Course Introduction and Overview

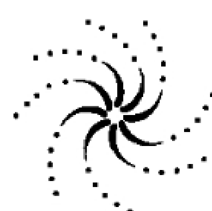
Centrifugal pumps are studied and designed based on both mechanical and hydraulic considerations. Examples of some of the mechanical aspects are vibration, seal compatibility, bearing selection, casing configuration, metallurgical suitability, and radial thrust and shaft deflection. Hydraulically the major subjects consist of head, capacity, hydraulic efficiency, power, and speed. With regard to total performance and efficient design, these two design branches are inescapably interdependent. The need for certain hydraulic performance can dictate mechanical design and conversely, mechanical design constraints can impact hydraulic attributes and performance. The scope of this course is limited to the study of the basic hydraulic parameters listed above for centrifugal pumps.

A brief discussion of the mathematics of variables and constants is undertaken to support the introduction and understanding of the Affinity Laws. The Affinity Laws allow Engineers to estimate the resulting changes to certain hydraulic parameters caused by the manipulation of a single centrifugal pump variable.

Centrifugal Pump Fundamentals



Pumps are broadly classified as kinetic or positive displacement. One of the sub-classifications of the kinetic pump branch is the centrifugal type. It consists of a "wet end" and a drive end. The wet end consists of a rotating impeller within a casing with inlet and outlet connections. It is coupled to either a constant or variable speed drive. Of all of the types of pumps, the centrifugal pump is the most commonly used. It has found favor because of its many advantages: simple construction, low relative cost, low maintenance, quiet operation, and reliability. Unfortunately, centrifugal pumps experience difficulty when handling viscous liquids and liquid/gas mixtures. Through the years an excellent knowledge base has been developed that allows for the accurate prediction of the hydraulic performance of centrifugal pumps.



Liquid is conveyed by the centrifugal pump by virtue of the kinetic energy imparted to the liquid by the rotating impeller. For a given diameter impeller at a given speed, a finite amount of energy (foot pounds) is transferred to each pound of liquid pumped regardless of the weight (density) of the liquid. This fact gives rise to the axiom that the resulting fluid height produced from this pumping operation, but not the pressure developed at the base of this fluid column, is identical irrespective of the liquid pumped. Liquid heights are referred to as *heads.* A pressure reduction occurs when the liquid moves from the pump inlet (suction connection) to the point at which it receives energy from the impeller. In pump hydraulics, suction refers to the inward movement of liquid through a conduit, such as a section of pipe, into the pump and ultimately to the eye of the impeller.

Suction is the negative pressure induced by the rotating impeller that draws the pumped liquid to a point such that energy may be imparted to it from the impeller vanes. The opposite of suction is discharge. The word suction is used as an adjective in many hydraulic terms, all of which of course refer to the inlet side of a pumping system.

SUMMARY

- Pumps are kinetic or positive displacement.
- The most prevalent kinetic pump type is the centrifugal.
- Efficient pump designs depend on mechanical and hydraulic considerations.

Basic Pump Parameters

The basic pump parameters can be subdivided into those that deal with purely hydraulic/liquid aspects and those that can be classified as more or less rotational in nature. This division has been made arbitrarily to facilitate a more simple method of learning for this course. For reasons that will become fully apparent, we will refer to the constituents of each subdivision as variables. In all, there are a total of six variables. Lets take a look at the two subdivisions, each of which has three variables.

Hydraulic Variables

The hydraulic variables consist of head, capacity (or flow), and efficiency. We will examine each one in detail:

Source: www.landstat.gsfc.nasa.gov

Head

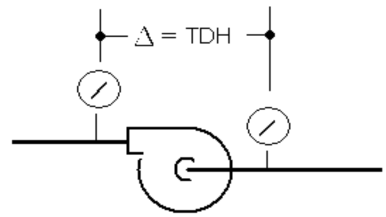
Although used extensively within the hydraulic engineering community, the term *head* is a somewhat archaic word whose etymology is from the Middle English. Its original meaning was literally a body of water kept in reserve at a height. Today the dictionary definition is:

(1) The difference in elevation between two points in a body of fluid; (2) the resulting pressure of the fluid at the lower point expressible as this height; broadly: the pressure of a fluid.

Head is simply a pressure unit that is commonly used in hydraulic engineering that is expressed in feet of pumped fluid. That is to say, it is the pressure that is exerted from the weight of a height of a given liquid; hence the unit of feet (meters in the metric system of units). There are numerous forms and references to hydraulic head, such as,

1. Fluid friction head;
2. Static suction head
3. Pump discharge head.

In this course we are normally dealing with a term more accurately referred to as Total Dynamic Head (TDH). Quite simply, this is the difference between the pressure on the discharge side of the pump and the pressure on the suction side. For a better understanding of hydraulic head, let's digress momentarily from the TDH concept and discuss what may be a more common consideration: pump discharge head. It is convenient to conceptualize discharge head by visualizing a single vertical pipe, infinitely long, connected to the outlet of a centrifugal pump. When operated, this pump's developed discharge pressure would "lift" the pumped liquid to an equilibrium height in the vertical pipe, identical to the pressure that would be produced by the weight of that same column of liquid. This particular height is known as *shut-off* head because it would simulate the head produced when the flow is zero, *e.g.*, against a closed valve. In the context of this course material however, the term head will normally be understood to mean total dynamic head. It will be denoted as H.



Capacity

The capacity of a pump is the amount of liquid conveyed per unit time. It is actually the volumetric rate of flow. Other common terms for capacity are flow rate and discharge rate. The classical English unit is gallons per minute (gpm). The metric equivalents are liters per minute (ℓ/min) or cubic meters per second (m^3/sec). Capacity will be denoted as Q.

Efficiency

In the real world, physical systems operate with inherent losses. What goes in does not necessarily come out. Efficiency is a measure or indication of the amount of loss. The term entropy is used to define unavailable or lost energy; entropy is ever increasing. We must be careful when we discuss efficiency because there are no less than four efficiencies involved in centrifugal pump systems. These are (1) hydraulic efficiency, (2) mechanical efficiency, and (3) drive efficiency. The overall pump operational efficiency (4) is the product of the three preceding efficiencies.

A complete discussion of hydraulic efficiency will be provided when the subject of power is undertaken. In the context of this course, efficiency means hydraulic efficiency and it will be denoted with the symbol E.

Mechanical efficiency is a measure of the losses between the drive output shaft and shaft input side of the impeller. For instance, frictional losses in couplings would be a contributing factor to lower mechanical efficiency. Relatively speaking, mechanical losses are small and are usually ignored. Drive efficiency refers to the effectiveness of the pump driver, be it either an electric motor, magnetic drive, or a steam turbine. As efficiencies go, electric motor efficiencies are extremely good and vary little with load or speed.

> **INTERESTING FACT**
> Pump hydraulic losses are attributable separately to the pumped liquid's viscosity and to numerous forms of internal cavitation. The more viscous the liquid the higher the hydraulic losses. Cavitation is the rapid formation and collapse of gas bubbles in the liquid associated with the rotation of the impeller.

Rotational Variables

The rotational (maybe they should be referred to as mechanical) variables are power, speed, and impeller diameter. We will examine each one in detail.

Power

In physics, power is defined as work per unit time. In the field of engineering, power is defined as the ability to do work. Units for power are the horsepower (hp) and the kilowatt (kw). With centrifugal pumps we deal with the former; the unit of horsepower is commonly used interchangeably with, and taken to mean the variable of power. Here again we must be careful. When we discuss horsepower there exists no less than three different horsepowers involved in centrifugal pump systems. These are (1) hydraulic horsepower, (2) brake horsepower, and (3) drive or motor horsepower. Hydraulic horsepower, sometimes referred to as water horsepower (*WHP*), is the power imparted to the liquid by the pump. It is defined by the following formula,

$$WHP = \frac{QH}{3960}$$

Where, Q = flow rate (capacity), gpm
H = head, feet of liquid

To provide a certain amount of power to the liquid a larger amount of power must be provided to the pump shaft to overcome inherent losses. The hydraulic efficiency is a measure of these losses and is the comparison of power input to the pump shaft to that of the poser transferred to the liquid. The power delivered to the pump shaft is known as break horsepower (*BHP*) and it is defined by the following formula,

$$BHP = \frac{QH}{3960E}$$

Where E = hydraulic efficiency, expressed a decimal fraction

The use of the abbreviation *BHP* for brake horsepower is relatively common. The quantity represented by *BHP* has been assigned to the mechanical power variable here for want of a better symbol. This was done with reluctance because in standard mathematical nomenclature it would imply that it is the produce of the separate variables: B, H, and P. In the instance it is not.

IMPORTANT FACT
The liquid specific gravity has been omitted in the power formulas presented. *BHP* varies directly with liquid specific gravity. A liquid with a S.G. of 1.5 would require 1.5X power.

Brake horsepower is the quantity that is generally provided by pump manufacturers on performance curves. For this reason, horsepower or simply power, shall be taken to mean brake horsepower in this course.

INTERESTING FACT

The term brake horsepower derives from the device that was originally used to measure it. The prony brake (dynometer) was invented by Gaspard deProny in 1821. It consists of an instrumented leather belt which provides friction against the rotating transmission shaft. The measured differential tension determines the work done and this value is multiplied by the measured shaft speed to arrive at the brake horsepower.

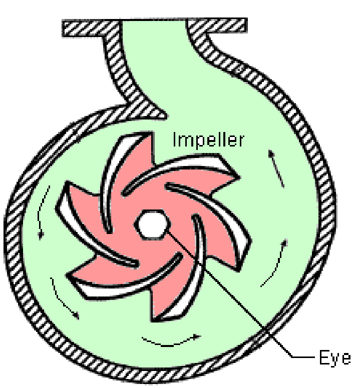
Prime movers, also known as drives, are machines that convert natural energy into work. Drive horsepower is the nominal or nameplate power rating of the prime mover. The two primary types of drives for centrifugal pumps are electric motors and steam turbines. The most popular type of electric motor in industrial applications is the three-phase induction motor, also called a *squirrel- cage* motor because of the appearance of its rotor. Standard motor horsepower ratings range from ⅛to 5,000 hp and greater. Turbines are classified descriptively by the type of driven apparatus, *e.g.*, generator-drive or mechanical-drive. Mechanical-drive (centrifugal pump) turbine horsepower ratings can range from a few horsepower to several thousand horsepower.

Rotation Speed

Rotational speed is the scalar quantity of the dynamics term known as angular velocity. Rotational speed is generally referred to simply as speed. The unit of revolutions per minute (rpm) is used in conjunction with speed. As stated above, centrifugal pumps are generally driven by AC induction motors and therefore have speed multiples at or near 60 depending on a consideration known as the slip factor. Variable frequency AC drives have gained in popularity and allow for variable pump speeds and improved efficiencies. Speed is denoted by the symbol *N*.

Impeller Diameter

Impeller diameter is the simplest variable to define. But first, let's discuss impellers. Impellers are essentially discs with outwardly radiating vanes which when rotated impart the motive centrifugal hydraulic force to the fluid being pumped. The impeller is mounted on the pump shaft and within the pump's casing. Centrifugal pump casings are fabricated to accept a given maximum impeller diameter allowing for a hydraulic clearance to facilitate fluid motion to the pump discharge point. Impellers are manufactured in many styles each with its own special attribute and function. Impeller diameter is 2X the distance of a line passing through the pump shaft center to the impeller periphery.



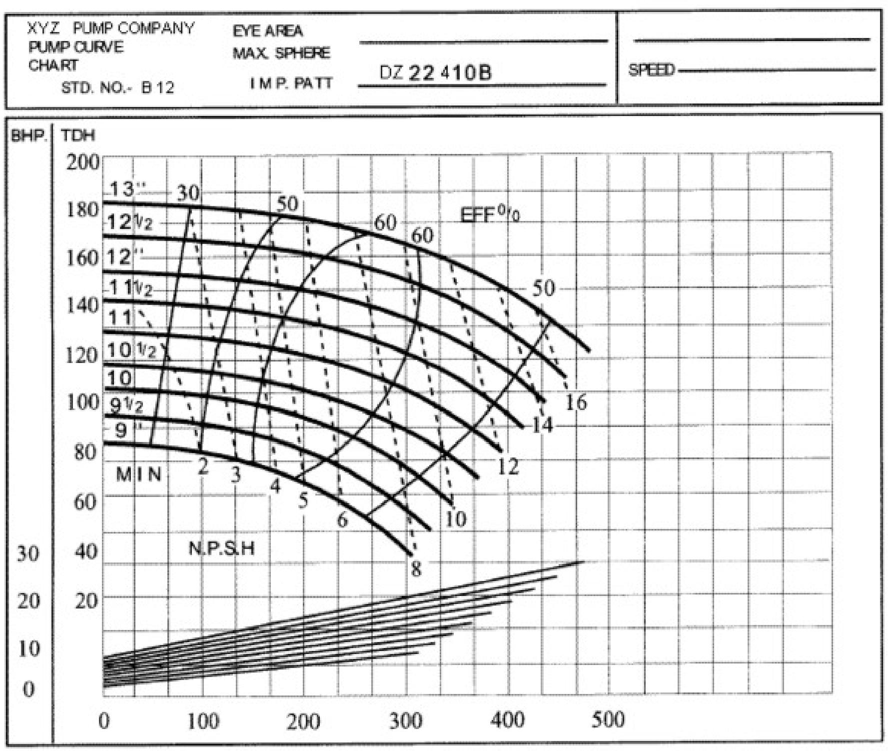
What is important about the diameter variable is that the peripheral velocity, which is directly proportional to the diameter, is directly relatable to the pump head developed. The impeller diameter is variable by virtue of the fact that the maximum dimension can be "turned down" or trimmed to produce a smaller diameter and thereby result in a lesser amount of centrifugal force delivered to the fluid. On the opposite hand, a pump operating with an undersized impeller can be retrofitted with a larger impeller up to the maximum that can be accommodated by the casing geometry.

Pump Performance Curves

A centrifugal pump's basic parameters are plotted on manufacturer's performance curves. The data generated is produced by conducting operational tests of the pump with water. The presentation of these parameters is somewhat standardized on a rectangular coordinate plane. The ordinate axis usually serves a dual role of head and brake horsepower while the abscissa accommodates the capacity. Performance curves are generated for constant speed and multiple impeller diameters represented as a family of essentially parallel decaying parabolic plots that characterize the relationship of head versus flow.

Other data presented on the curves are:

1. Efficiency:
2. BHP;
3. $NPSH_R$



Lines of constant efficiency are plotted on manufacturer's pump performance curves as superimposed incomplete parabolic arcs generally opening in the positive direction. An rough estimate of the efficiency value using these plots is sometimes the best that can be accomplished from the pump performance curve.

Brake horsepower is usually plotted near the bottom of the manufacturer's pump performance curves and is represented by a group of straight lines with positive slope. While not normally labeled as such, each straight line plot corresponds to an impeller diameter in the same order that they are presented for the impeller characteristic curves. Because the value of *BHP* can be more accurately read from the chart than the efficiency value, many Engineers prefer to back-calculate the pump efficiency at a specific head and flow rate by using this value of *BHP*, and substituting it into a rearranged power equation that would appear as,

$$E = \frac{QH}{3960 BHP}$$

Look at the example of a typical manufacturer's pump performance curve for a constant speed pump on the previous page and see if you can find each of the variable plots just mentioned.

The other major type of performance curve is of the case of a fixed impeller diameter operating at multiple speeds. It provides much of the same information and appears much like the constant speed curve except multiple characteristic curves representing various speeds are provided in lieu of the family of impeller curves.

SUMMARY

- Performance curves plot or provide information for all of the pump variables.
- Performance curve presentations exist for multiple impeller/constant speed and fixed impeller diameter/variable speed.
- Performance curves are generated from tests using water.

The Affinity Laws

The affinity laws are mathematical relationships that allow for the estimation of changes in pump performance as a result of a change in one of the basic pump parameters (variables). As such, it is important to understand the concept of variables.

A Brief Math Primer

Mathematical relationships can consist of variables and constants. As their names imply, variables change and constants remain unchanged. Variables can be either independent or dependent. In other words, a purposeful change in an independent variable results in a corresponding change in the other variable(s) whose value is dependent on the defined mathematical relationship. It is convenient to think of (and group) the independent variable(s) and constants on the right side of the mathematical relationship, *i.e.*, the formula or equation, and to isolate a single dependent variable on the left side. A classical example could look something like,

$$y = x^2 + 3x - 6$$

This expression in essence says the value of y is dependent upon the independently assigned values of x by a defined quadratic relationship consisting of a subtractive constant.

The Affinity Laws viewed as Variables and Constants

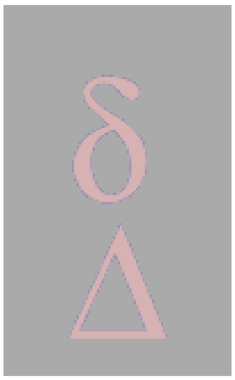
Now, with a slight departure from the standard narrative presented above, let's look at:

1. Holding a single pump parameter constant and,
2. Manipulating a given pump variable and,
3. Observing how this change affects the remaining variables.

Before we begin, understand that the definition of the word *law* is pretty strong. In its simplest form, the term *law* means a principle that has been proven to be true for all cases. As we will learn, the affinity laws have limitations. Maybe a better term would be *Affinity Guides* or *Affinity Postulates*; at any rate, we will accept the commonly used term and will find important uses for the concepts it represents.

Why Change Pump Variables?

Pump variables are changed to change pump performance. For example, it would make little sense to completely replace a given pump in order to simply reduce the head in a system. It may be more efficient to simply "turn-down" the impeller, *i.e.*, change the diameter variable, to produce the desired result. In so doing, we would want to know how the flow rate may be affected with this new impeller diameter. With the advent of the variable speed drive, a pump's rotational speed is easily and conveniently adjusted over a broad range. This is an excellent method of controlling the flow rate of material streams in processes that require variability. What changes in head and flow might occur with the manipulation of a pump's speed using a variable drive? In another example, it may become necessary to estimate the performance of a centrifugal pump whose impeller diameter or speed may not be indicated on a standard performance curve. The approximate curves for a new impeller diameter or new speed could be determined by means of the affinity laws. In short, the affinity laws can come to the rescue of an enterprising Engineer.



ADDITIONAL INFORMATION
Because affinity means an inherent similarity between things, the affinity laws are often referred to as the *pump similarity laws*.

There are two sets of affinity laws: (1) A set that is based on the impeller diameter variable assigned as a constant ($D = c$) and (2) a set that is based on the speed variable as a constant ($N = c$). Let's take a look at each set separately.

The premise of the first set of affinity laws is:

> *For a given pump with a fixed diameter impeller, the capacity will be directly proportional to the speed, the head will be directly proportional to the square of the speed, and the required power will be directly proportional to the cube of the speed.*

While not difficult, the derivation of the following proportionalities and resulting equations is beyond the scope of this course. The affinity laws premise stated above can be expressed as,

Set 1 Affinity Laws Premise:
(for D = constant, E = constant)*

$$Q \propto N \quad ; \qquad H \propto N^2 \quad ; \qquad BHP \propto N^3$$

* Usually there is no appreciable change in efficiency within the range of normal pump operational speeds. For this reason, the Set 1 Affinity Laws can be considered reasonably reliable and accurate.

These proportionalities give rise to the relationships:

Set 1 Affinity Laws:

$$\left\{ \frac{Q_1}{Q_2} = \frac{N_1}{N_2} \quad ; \quad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \quad ; \quad \frac{BHP_1}{BHP_2} = \left(\frac{N_1}{N_2}\right)^3 \right\}$$

Where the symbols are as previously defined and the subscripts "1" and "2" can be thought of as *old* and *new* or alternatively *before* and *after*, respectively.

Returning to our math primer, we learned that by separating and grouping variables as independent and dependent, we can sometimes improve the usability of mathematical presentations. Rearranging the above expressions result in,

Set 1 Affinity Laws:

$$\left\{ Q_2 = Q_1\left(\frac{N_2}{N_1}\right) \quad ; \quad H_2 = H_1\left(\frac{N_2}{N_1}\right)^2 \quad ; \quad BHP_2 = BHP_1\left(\frac{N_2}{N_1}\right)^3 \right\}$$

Whereby the normally and usually sought after variables Q_2, H_2, and BHP_2 conveniently become the dependent variables.

INTERESTING FACT

The hydraulic variable of head *(H)* stated in the affinity laws is normally taken to be total dynamic head. However, the laws hold for *H* to be any centrifugal pump hydraulic head. Allowable substitutions could be $NPSH_R$, velocity head, *et.al.*

Before we get too far, let's take a look at an example that illustrates the use of the Set 1 Affinity Laws.

EXAMPLE 1: (Set 1 Affinity Laws)

Problem: A centrifugal pump equipped with a variable frequency (speed) drive running at 3500 rpm is discharging 240 gallons per minute corresponding with a head of 287 feet. The horsepower is 35.5. If the pump's speed is reduced to 2900 rpm, what will be the revised flow rate, head, and power required.

Solution:

(1) Since there is no reference to the impeller diameter variable, it is safe to assume that D = constant and that the Set 1 Affinity laws are applicable and can be applied.

(2) Given, in order, are: N_1 = 3500 rpm, Q_1 = 240 gpm, H_1 = 287 feet, and BHP_1 = 35.5 hp.

(3) Using the Set 1 Affinity Laws find:

Revised flow rate:

$$Q_2 = Q_1\left(\frac{N_2}{N_1}\right) = 240\left(\frac{2900}{3500}\right) = 199 \text{ gpm}$$

Revised head:

$$H_2 = H_1\left(\frac{N_2}{N_1}\right)^2 = 287\left(\frac{2900}{3500}\right)^2 = 197 \text{ feet}$$

Revised power:

$$BHP_2 = BHP_1 \left(\frac{N_2}{N_1} \right)^3 = 35.5 \left(\frac{2900}{3500} \right)^3 = 20.2 \text{ hp}$$

Now, let's examine the second set of affinity laws. This set considers a centrifugal pump's performance variations with changes in impeller diameter while assigning the rotational speed as the constant. We will call this group, Set 2. There is a caveat with this set however. The laws relating to the impeller diameter assigned as the independent variable are not as accurate as those relating to the independent variable of rotational speed. As has been stated earlier, in most practical applications, the impeller is trimmed or "turned-down" to a smaller diameter, leaving the remaining pump geometry unchanged. The Set 2 Affinity Laws are not as accurate as Set 1 because the dimensions of the pump are not changed in proportion to the change in the impeller diameter variable. This fact can cause a change in the original efficiency. With that said, the Set 2 Affinity Laws are sufficiently accurate enough to allow for reasonable predictions of performance to be made.

The premise of the second set of affinity laws is:

> *For a given pump with a constant speed, the capacity will be directly proportional to the impeller diameter, the head will be directly proportional to the square of the impeller diameter, and the required power will be directly proportional to the cube of the impeller diameter.*

<u>Set 2 Affinity Laws Premise:</u>
(for N = constant, E = constant)*

$$Q \propto D \quad ; \qquad H \propto D^2 \quad ; \qquad BHP \propto D^3$$

* Usually there <u>are</u> changes in efficiency associated with changes in impeller diameter and as stated above, inaccuracies can be introduced into the predictions.

These proportionalities give rise to the relationships:

<u>Set 2 Affinity Laws:</u>

$$\left\{ \frac{Q_1}{Q_2} = \frac{D_1}{D_2} \quad ; \qquad \frac{H_1}{H_2} = \left(\frac{D_1}{D_2} \right)^2 \quad ; \qquad \frac{BHP_1}{BHP_2} = \left(\frac{D_1}{D_2} \right)^3 \right\}$$

Rearranging the above expressions for convenience results in,

<u>Set 2 Affinity Laws:</u>

$$\left\{ Q_2 = Q_1 \left(\frac{D_2}{D_1} \right) \quad ; \qquad H_2 = H_1 \left(\frac{D_2}{D_1} \right)^2 \quad ; \qquad BHP_2 = BHP_1 \left(\frac{D_2}{D_1} \right)^3 \right\}$$

Let's take another hypothetical example as an illustration of the use of the Set 2 Affinity Laws.

EXAMPLE 2: (Set 2 Affinity Laws)

Problem: A 4 x 3 –13 1750 rpm centrifugal pump equipped with an 11 inch diameter impeller requires 14.3 bhp when delivering 300 gpm at a head of 111 feet. Predict the new capacity, head, and power required when the impeller diameter is reduced to 9 inches.

ADDITIONAL INFORMATION

While there is no standardized system of specifying centrifugal pump size, one popular method is to first indicate the nominal suction size (inches), then the nominal discharge size (inches), followed by the maximum diameter impeller (inches) that the pump casing can accommodate. Example: 3 x 2 -10

Solution:

(1) Since two impeller diameters are stated and there is no reference to a speed change, it is safe to assume that N = constant and that the Set 2 Affinity laws are applicable and can be cautiously applied.

(2) Given, in order, are: N_1 = 1750 rpm, D_1 = 11 inches, BHP_1 = 14.3 hp, and Q_1 = 300 gpm, H_1 = 111 feet.

(3) Using the Set 2 Affinity Laws find:

Approximate new flow rate:

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right) = 300\left(\frac{9}{11}\right) = 246 \text{ gpm}$$

Approximate new head:

$$H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 = 111\left(\frac{9}{11}\right)^2 = 74 \text{ feet}$$

Approximate new power:

$$BHP_2 = BHP_1\left(\frac{D_2}{D_1}\right)^3 = 14.3\left(\frac{9}{11}\right)^3 = 7.8 \text{ hp}$$

SUMMARY

- It is convenient to consider the basic parameters as variables.
- These variables are changed to change pump performance.
- Changes in pump performance can be predicted by the affinity laws.

Course Summary

Centrifugal pump performance, efficiency, and reliability depend on equitable consideration to all pump design aspects. There are many facets that contribute to the creation of an optimum centrifugal pump design. Pump designers must take into consideration both the mechanical and hydraulic aspects and limitations presented by a set of application criteria.

Processes require the satisfaction of the hydraulic parameters of head and capacity and this is sometimes accomplished by the manipulation of the pump's speed and less frequently with the physical modification of the impeller. Affinity laws are a series of ratios that enable the Engineer to predict performance as pump conditions change. The "laws" have proven to be an excellent tool for estimating or predicting a given centrifugal pump's revised hydraulic performance after changes are made to one of the basic pump parameters.