



PDHonline Course M239 (4 PDH)

Introduction to Material and Energy Balance

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Introduction to Material and Energy Balance

Course Content

A material balance in its most broad definition is the application of the law of conservation of mass, which states matter is neither created nor destroyed. Matter may flow through a control volume and may be reacted to form another species, however, no matter is ever lost or gained. The same is true for energy. As with material balances, we can apply the law that energy is neither created nor destroyed, it is simply converted into another form of energy. The law of conservation of mass and energy leads to what is called a mass (material) and energy balance.

Law of Conservation of Mass

The law of conservation of mass states that mass can neither be created nor destroyed. Thus in a processing plant, the total mass of material entering the plant must equal the total mass of material leaving the plant, less any accumulation left in the plant. If there is no accumulation, then the simple rule holds that "what goes in must come out".

For example, if milk is fed into a centrifuge to separate it into skim milk and cream, under the law of conservation of mass, the total number of kilograms of material (milk) entering the centrifuge per minute must equal the total number of kilograms of material (skim milk and cream) that leave the centrifuge per minute. Similarly, the law of conservation of mass applies to each component in the entering materials.

The law of conservation of mass will apply to each component, unit operation and the entire process. Thus in each area of the process:

$$\text{Mass In} = [\text{Mass Out} + \text{Mass Stored}]$$

$$\text{Raw Materials} = \text{Products} + \text{Wastes} + \text{Stored Materials}$$

Note that the wastes include the losses which are unidentified materials. This is especially true if there are chemical changes occurring in the plant operations. For example, in a plant that produces sugar, if the total quantity of sugar cane going into the plant is not equaled by the total of the purified sugar and the sugar in the waste liquors, then there is something wrong. Sugar is either being burned (chemically changed) or getting accumulated in the plant or else it is going unnoticed down the drain somewhere. So the material balance equation can be modified as:

$$\text{Raw Materials} = \text{Products} + \text{Waste Products} + \text{Stored Products} + \text{Losses}$$

Before we proceed further on developing the material and energy balances, let's first discuss briefly some important terms:

Classification of Processes

A. Based on how the process varies with time.

- a. Steady-state process is one where none of the process variables change with time. Every time we take a snapshot, all the process variables have the same values as in the first snapshot.
- b. Unsteady-state (Transient) process is one where the process variables change with time. Every time we take a snapshot, many of the variables have different values than in the first snapshot. (One class of unsteady-state processes is oscillatory, where the process variables change with time in a regular way. All other unsteady processes may be called Transient meaning that the process variables continuously evolve over time).

B. Based on how the process was built to operate.

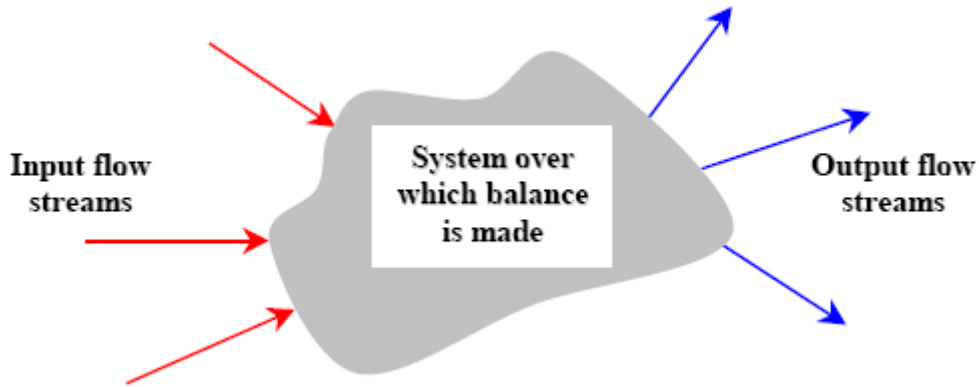
- a. A Continuous process is one that has the feed streams and product streams moving into and out of the process all the time. Examples are an oil refinery, a power grid, pool filter and a distillation process.
- b. A Batch process is a process one, where the feed streams are fed to the process to get it started. The feed material is then processed through various process steps and the finished products are taken out at specific times. Steps:
 - Feed is charged into vessel
 - Process is started
 - No mass is added or removed from vessel (temperature and pressure are usually monitored and controlled)
 - At some conditions or fixed time, products are removed

In a batch process no material is exchanged with the surroundings during the process. Examples: baking cookies, fermentations, small-scale chemicals (pharmaceuticals)

- c. A Semi-batch process (also called semi-continuous) is a process that has some characteristics continuous and batch processes. Examples are washing machine, fermentation with purge etc.

MATERIAL BALANCES

A material balance (also called a mass balance) is an accounting of material entering and leaving a system. Material balance can be applied to entire process or any unit operation. Whatever its nature, the input flow streams (mass and energy) always balance with the output flow streams (mass and energy).



The Material Balance Equation

The fact that matter and energy cannot be lost nor gained can be extrapolated into the basic, most general form of the equation, which is as follows:

$$\text{INPUT} - \text{OUTPUT} = \text{ACCUMULATION}$$

If the process is at steady-state, there is no accumulation of mass within the process. Thus

$$\text{INPUT} = \text{OUTPUT}$$

When we apply this equation to a process, it is best to write it as

$$\Sigma \text{Masses entering via feed streams} = \Sigma \text{Masses exiting via product streams}$$

We understand that we must include *the mass of every element in every stream*. The above equation can be applied to batch and continuous processes as

$$\Sigma \text{Mass in} = \Sigma \text{Mass out} \dots \dots \dots \text{for a batch process, and}$$

$$\Sigma \text{Mass in by flow} = \Sigma \text{Mass out by flow} \dots \dots \dots \text{for a continuous process.}$$

When mass balances are written for specific process involving chemical reaction(s), we must account for the formation of product chemicals and the consumption of feed chemicals. We must remind ourselves that the law of conservation of mass means total mass. For this case, we must write a mass balance for each chemical and account its formation and consumption as follows

$$\Sigma \text{Mass in} + \text{Mass formed by reaction} = \Sigma \text{Mass out} + \text{Mass used by reaction}$$

Or, written more simply as

$$\text{Input} + \text{generation} = \text{output} + \text{consumed}$$

The generation term may then describe chemical reaction rates, which might be positive or negative, just as for accumulation.

Types of Mass Balances

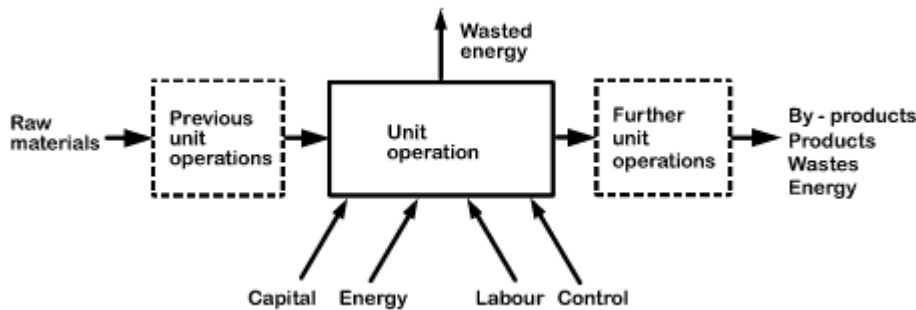
Mass balances are either ‘integral mass balances’ or ‘differential mass balances’.

- a. Differential Balance is a balance taken at one particular instant in time. It is generally applied to a continuous process. If the process is at steady state, a differential balance applied at any time gives the same result. Each term in a differential balance represents a process stream and the mass flow rate of the material(s) (i.e. it uses mass per time [kg/s] and not mass).
- b. Integral balance is a balance taken at two specific instants in time or the entire time of the process at once. It describes what has happened over the time period between the two points. An integral balance is generally applied to the beginning and the end of a batch process and it accounts for what happens to the batch of material. Therefore an integral balance is a simplistic option for batch processes but may also be used for continuous or semi-batch processes when it is integrated from t_1 to t_2 (typically very difficult). Each term in an integral balance represents a process stream and the mass of the material(s) (so it uses amounts rather than rates: e.g., mass NOT mass/time).

An integral mass balance is a black box approach which focuses on the overall behavior of a system whereas a differential mass balance focuses on mechanisms within the system (which in turn affect the overall behavior). To make a differential mass balance one must also describe the interior of the system. It is generally applied to a continuous process.

What balances can one write?

A process can be viewed overall or as a series of units. Each unit is a unit operation that can be represented by a box as shown in figure below:



Overall View of an Engineering Process

Into the box go the raw materials and energy; out of the box come the desired products, by-products, wastes and energy. The mass flow in and out of a control box (through a physical or virtual boundary) must be equal. The equipment within the box will enable the required changes to be made with as little waste of materials and energy as possible. *In other words, the desired products are required to be maximized and the undesired by-products and wastes minimized.* Control over the process is exercised by regulating the

flow of energy, or of materials, or of both. The material balances are required to be developed at the various levels.

- 1) Overall Material balance: This involves the input and output streams for complete plant.
- 2) Section wise Material balances: This involves M & E balances to be made for each section/ department/cost center. This would help to prioritize focus areas for efficiency improvement.
- 3) Equipment-wise Material balances: Material balances, for key equipment would help assess performance of equipment, which would in turn help identify and quantify energy and material avoidable losses.

The choice among the above depends on the reasons for making the balance and on the information that is required. A major factor in industry is, of course, the value of the materials and so expensive raw materials are more likely to be considered than cheaper ones, and products than waste materials. Note that when developing material balance equations, there is no concern with the internal details, but with only the passage of a material across a volume's boundaries.

Material Balance Procedure

The first step is to look at the three basic categories: materials in, materials out and materials stored. Then the materials in each category have to be considered whether they are to be treated as a whole, a gross mass balance, or whether various constituents should be treated separately and if so what constituents. To take a simple example, it might be to take dry solids as opposed to total material; this really means separating the two groups of constituents, non-water and water. Typical steps are as follows:

- 1) Define basis & units: In order to develop a material balance, a basis for the balance must be defined, as well as the system that is being analyzed.
- 2) Draw a flowchart: A boundary must be established, so the flow streams in and out can be determined.
- 3) Do the degree of freedom analysis: It provides a mechanism to check whether the material balance equations are solvable. Typically the equations are solvable if the degree of freedom is 1.
- 4) Write material balance equations: Finally write the material balance equations.

BASIS & UNITS

Having decided which constituents need consideration, choose a basis of calculation on amount or flow rate of one of the process streams. This might be some mass of raw material entering the process in a batch system, or some mass per hour in a continuous process. In continuous processes, time enters into consideration and the balances are related to unit time. For example in considering a continuous centrifuge separating whole milk into skim milk and cream, if the material holdup in the centrifuge is constant both in mass and in composition, then the quantities of the components entering and leaving in the different streams in unit time are constant and a mass balance can be written on this basis. Such an analysis

assumes that the process is in a steady state, i.e. flows and quantities held up in vessels do not change with time. Sometimes it is unimportant what basis is chosen and in such cases a convenient quantity such as the total raw materials into one batch or passed in per hour to a continuous process is often selected. Having selected the basis, then the units may be chosen such as mass, or concentrations which can be expressed in many ways: weight/ weight (w/w), weight/volume (w/v), molar concentration (M), mole fraction.

- ❖ The weight/weight concentration is the weight of the solute divided by the total weight of the solution and is the fractional form of the percentage composition by weight.
- ❖ The weight volume concentration is the weight of solute in the total volume of the solution. With gases, concentrations are primarily measured in weight concentrations per unit volume, or as partial pressures.
- ❖ The molar concentration is the number of molecular weights of the solute expressed in kg in 1 m^3 of the solution.
- ❖ The mole fraction is the ratio of the number of moles of the solute to the total number of moles of all species present in the solution.

Units

We use values, units and dimensions all the time:

- 1) **Value:** A value is the numerical quantity ; for example: 5.2
- 2) **Units:** The units tell what that quantity represents; for example: 5.2 liters.
- 3) **Dimensions:** The dimensions are the measurable properties that the units represent; for example: a liter is a unit of volume (units are a specific example of a dimensional quantity).

To distinguish value, units and dimensions, let's consider two examples:

- A carton of milk has value = 1; units = carton and dimensions = volume
- 1/4 pound burger has value = 0.25; units = pounds and dimensions = mass

In applications of mathematics it is usually very important to keep track of dimensions or physical units such as feet, volts, dollars, etc. In pure mathematics the sum of 15 and 2 is 17 and that's the end of it. However, units can completely change arithmetically bizarre equations such as $15 + 2 = 1$ or $15 + 2 = 7$ into correct statements. They can make sense in the context of money and ordinary measurement $15 \text{ cents} + 2 \text{ nickels} = 1 \text{ quarter}$ or $15 \text{ ft} + 2 \text{ yd} = 7 \text{ yd}$.

So never neglect dimensions or physical units. For example, each of the above equations would be written in the same units -- $3 \text{ nickels} + 2 \text{ nickels} = 5 \text{ nickels}$ or $15 \text{ ft} + 6 \text{ ft} = 21 \text{ ft}$, so that the units on the left side of the equation match the units on the right, and we end up with our usual arithmetic. Units should never be mixed in scientific formulas.

Let's take another very tricky example which does not sound as if it has anything to do with units, yet that is one way to deal with it.

Example

Suppose it takes one man 5 hours to paint a house, and it takes another man 3 hours to paint the same house. If the two men work together, how many hours would it take them?

Solution

The first thing to notice is that if the fast painter can paint a house alone in three hours, the job will be completed *sooner* than three hours if the slow painter (five hours per house) joins him. We might estimate the time by first averaging the painters' hours, $(3+5)/2 = 4$ hrs. Then, since the two of them are both painting, divide this average by 2. So the answer should be *about two hours*. Now let's do it via units, a sort of dimensional analysis:

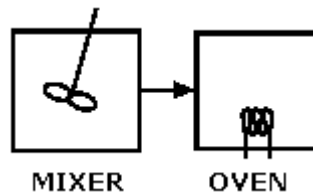
- Fast painter -- 3 hrs / house or 1/3 house per hr = 1/3 house/hr
- Slow painter -- 5 hrs / house or 1/5 house per hr = 1/5 house/hr
- Combined -- 1/3 house/hr + 1/5 house/hr = $(1/3 + 1/5)$ house/hr = 8/15 house/hr

This translates to 15/8 hrs/house or 15/8 hours per house. So it will take them 15/8 = 1.875 hours, or one hour and 52 minutes to paint the house (1:52:30, if you think 30 sec is significant!).

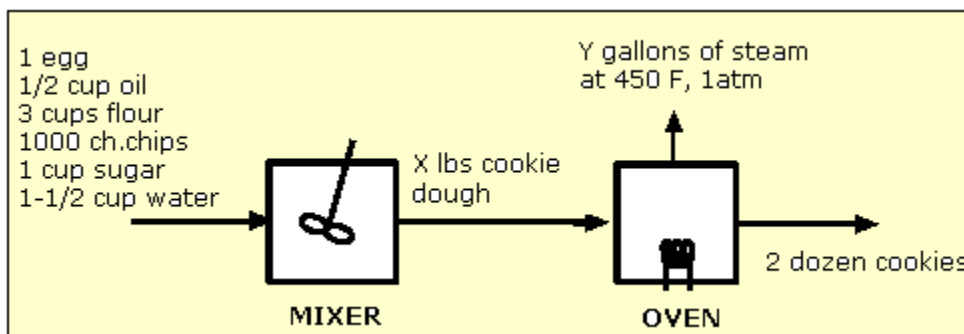
FLOWCHART

Flow charts is a schematic representation of the process, involving various process units -input resources, conversion steps and output and recycle streams. So what are process units or streams?

Consider a bakery example; in its simplest form, a bakery must consist of two steps: a mixing step, and a cooking step. You might represent these two steps like this:



We are now on our way to our first flowchart. The above representation is a plain schematic of the process that does not highlight the process stream. A process stream is a line which represents the movement of material to/from process units. Typically these streams are labeled with information regarding the amounts, compositions, temperatures, pressures, etc. of the components. So if we now add process streams to our diagram above, we will have a valid flowchart.



Keys to Developing Successful Flowchart

The identification and drawing up a unit operation/process is prerequisite for energy and material balance.

- 1) Draw a good labeled diagram: Draw boxes (or other symbols) for all equipment and arrows indicating all streams and their directions of flow. Standard practice is to show the process flows from left to right. All equipment and all streams should be named or labeled with all the known information about the streams.
- 2) Write values of all known stream variables (units) such as flow rates, compositions, temperature and pressure. Use common basis (mass or moles – convert volumes). All values should be on the calculation basis you choose. Scale the flowchart if needed i.e. change all stream amounts or rates by a proportional amount. Remember that compositions should remain unchanged. If you scaled the flowchart to a different basis than stated in the problem, do not forget to re-scale.

Much of the difficulty experienced with material balances can be traced back to an incomplete diagram or ambiguous symbols/units. Put all known material flow with units on the diagram. Define and label with units all unknowns. Use x , y , z with proper subscripts, e.g. x to denote mass or mol fraction (be sure to state which one). An unknown symbol without defining units is not useful.

DEGREE OF FREEDOM ANALYSIS

When attempting to solve a material balance problem for unknown, one should first ask 3 questions:

- 1) Is the problem solvable with the information given?
- 2) How many equations do I need?
- 3) Where do they come from?

Analyzing and answering these questions can be done by performing a degree-of-freedom analysis. The degree of freedom is defined by equation:

$$\text{Degrees of Freedom} = \text{Unknowns} - \text{Equations}$$

If the degree of freedom is zero, then the problem is said to be correctly specified – the number of unknowns is equal to the independent equations and problem can be solved.

- 1) If the degree of freedom is positive, then the number of unknowns is more than independent equations and the problem is said to be underspecified. It is not possible to solve for all of the unknown stream variables. This may be due to that the relations have been overlooked.
- 2) If the degree of freedom is negative, then the problem is said to be over specified and redundant (possibly inconsistent) information needs to be discarded before a unique solution may be determined.
- 3) If the degree of freedom is 1, you may assume a basis of calculation

Steps to Performing Degree-of Freedom Analysis

When given a material balance problem:

- 1) Draw and completely fill and label flowchart.
- 2) Count the unknown variables in the flowchart
- 3) Count independent equations relating them.
- 4) See if the number of unknowns and equations are equal

Rule: *A problem is solvable, if the number of equations is equal to the number of unknowns i.e. when degrees of freedom =0.*

WRITING MATERIAL BALANCE EQUATIONS

Follow the steps below:

- Express what the problem is asking you to calculate. (you must know what the final destination is)
- Write balances first that involve the fewest unknown variables.
- Convert additional relations between variables into equations
- Solve set of equations and write results on flow chart
- Scale the results if answers are needed on a different basis

A very simple example:

A class room has 50 students. Some study engineering and some study science. The number of science students is four times the number of engineering students. There are 10 engineering students in the class. Therefore putting this information in form of equations:

$$\begin{array}{l}
 S + E = 50 \\
 S = 4E \\
 E = 10
 \end{array}
 \left. \vphantom{\begin{array}{l} S + E = 50 \\ S = 4E \\ E = 10 \end{array}} \right\} \Rightarrow \left. \begin{array}{l} S - 4E = 0 \\ 5E = 50 \end{array} \right\} \Rightarrow S + E = 50$$

Let's check some more examples:

EXAMPLES

Example #1

A solution which contains 10 % solids is mixed with a 25 % solid solution. A single output which is 20 % solid is removed. If the 10 % solution enters at 5.0 kg/s, what are the other rates? (Assume no accumulation)

Basis:

Solution A (INPUT) = 5 kg/s

Solution B (INPUT) = X kg/s

Solution C (OUTPUT) = Y kg/s

Therefore A + B = C

1) Total Equation: $5 + X = Y$EQ-1

Since solution A contains 10% solids; solution B contains 25% solids and solution C contains 20% the equation can be written as:

2) Solids: $0.1 * 5 + 0.25 * X = 0.2 * Y$EQ -2

Similarly a second equation can be written for liquid component:

3) Liquids: $0.9 * 5 + 0.75 * X = 0.8 * Y$EQ -3

Substituting value of Y from EQ-1 in EQ-2

$$0.5 + 0.25 X = 0.2 * (5 + X)$$

$$0.05 X = 0.5$$

$$X = 10 \text{ kg/s}$$

Substituting X in EQ-1

$$Y = 15 \text{ kg/s}$$

Example #2

A solution which is 80 % oil, 15 % usable by-products, and 5 % impurities, enters a refinery. One output is 92 % oil and 6 % usable by-products. The other output is 60 % oil and flows at the rate of 1000 L/h. (assume no accumulation, percents by volume)

- What is the flow rate of the input?
- What is the percent composition of the 1000 L/h output?

- What percent of the original impurities are in the 1000 L/h output?

Basis:

INPUT Stream A = X kg/s

OUTPUT STREAM B = Y L/hr

OUTPUT STREAM C = 1000 L/hr

Material Balance Equations

1) Total: $X = Y + 1000$EQ -1

2) Oil: $0.8 * X = 0.92 * Y + .6 * 1000$EQ -2

3) UBP: $0.15 * X = 0.06 * Y + v * 1000$EQ -3

4) IMP: $0.05 * X = 0.02 * Y + w * 1000$EQ -4

5) OUTPUT impurities & UBP: $v + w = 0.4$EQ-5

Solving Equations 1 and 2; substituting value of X in EQ-2

$$0.8 (Y + 1000) = 0.92 * Y + 600$$

$$0.8 Y + 800 = 0.92 Y + 600$$

$$Y = 1666 \text{ L/hr}$$

Substituting Y in EQ-1

$$X = 1666 + 1000 = 2666 \text{ L/hr}$$

Thus the flow rate of input stream is 2666 L/hr.

Substituting value of X and Y in EQ-3; $v = 30\%$

Substituting value of v in EQ-5; $w = 10\%$

Thus the composition of 1000 L/hr output stream is 60% oil, 30% usable by products and 10% impurities.

$$\text{Impurities in input stream} = 0.05 * 2666 = 133.3 \text{ L/hr}$$

$$\text{Impurities in 1000 L/hr stream (10\%)} = 100 \text{ L/hr}$$

$$\text{Therefore impurities in 1000 L/hr stream as percentage of input stream} = 100/133.3 = 75\%.$$

Example # 3: Constituent balance

Skim milk is prepared by the removal of some of the fat from whole milk. This skim milk is found to contain 90.5% water, 3.5% protein, 5.1% carbohydrate, 0.1% fat and 0.8% ash. If the original milk contained 4.5% fat, calculate its composition assuming that fat only was removed to make the skim milk and that there are no losses in processing.

Basis: Consider 100 kg of skim milk output

INPUT whole milk = X kg

OUTPUT skim milk = 100 kg

OUTPUT Fat = Y kg

Material Balance Equations

1) Total: $X = Y + 100$EQ -1

2) Fat: $0.045 * X = 0.001 * 100 + Y$EQ -2

Substituting Y from EQ -1 to EQ - 2

$0.045 X = 0.1 + X - 100$ or

$0.955 X = 99.9$

$X = 104.6$ kg

Substituting X in EQ -1

$Y = 4.6$ kg

So the composition of whole milk:

Water = $90.5 / 104.6 = 86.5\%$

Protein = $3.5 / 104.6 = 3.3\%$

Carbohydrate = $5.1 / 104.6 = 4.9\%$

Ash = $0.8 / 104.6 = .76\%$

Example #4: Continuous Process

In a continuous centrifuging of milk, if 35,000kg of whole milk containing 4% fat is to be separated in a 6 hour period into skim milk with 0.45% fat and cream with 45% fat, what are the flow rates of the two output streams from a continuous centrifuge which accomplishes this separation?

Basis: 1 hour's flow of whole milk

Total mass input per hour = $35000/6 = 5833$ kg

Total mass output for skim milk = Y

Total mass output for cream = Z

Material Balance Equations

1) Mass In = Mass Out: $5833 = Y + Z$EQ -1

2) Fat In = Fat Out: $0.04 * 5833 = 0.0045 * Y + .45 * Z$EQ - 2

Substituting Z from EQ -1 to EQ -2

$0.04 * 5833 = 0.0045 Y + 0.45 * (5833 - Y)$

$233 = 2625 - 0.446Y$

$Y = 5363 \text{ kg}$

Substituting Y in EQ - 1

$Z = 470 \text{ kg}$

So that the flow of skim milk is 5363 kg / hr and skim milk $(5833 - 5363) = 470 \text{ kg/hr}$

The time unit has to be considered carefully in continuous processes as normally such processes operate continuously for only part of the total factory time. Usually there are three periods, start up, continuous processing (so-called steady state) and close down, and it is important to decide what material balance is being studied. Also the time interval over which any measurements are taken must be long enough to allow for any slight periodic or chance variation.

In some instances a reaction takes place and the material balances have to be adjusted accordingly. Chemical changes can take place during a process, for example bacteria may be destroyed during heat processing, sugars may combine with amino acids, fats may be hydrolyzed and these affect details of the material balance. The total mass of the system will remain the same but the constituent parts may change, for example in browning the sugars may reduce but browning compounds will increase.

Example #5: Concentrations

A solution of common salt in water is prepared by adding 20 kg of salt to 100 kg of water, to make a liquid of density 1323 kg/m³. Calculate the concentration of salt in this solution as a (a) weight fraction, (b) weight/volume fraction, (c) mole fraction, (d) molal concentration.

(a) Weight fraction:

$20 / (100 + 20) = 0.167$: % weight / weight = 16.7%

(b) Weight/volume:

A density of 1323kg/m³ means that 1m³ of solution weighs 1323kg, but 1323kg of salt solution contains

$(20 \times 1323 \text{ kg of salt}) / (100 + 20) = 220.5 \text{ kg salt} / \text{m}^3$

1 m³ solution contains 220.5 kg salt.

Weight/volume fraction = $220.5 / 1000 = 0.2205$

And so weight / volume = 22.1%

c) Moles of water = $100 / 18 = 5.56$

Moles of salt = $20 / 58.5 = 0.34$

Mole fraction of salt = $0.34 / (5.56 + 0.34) = 0.058$

d) The molar concentration (M) is $220.5/58.5 = 3.77$ moles in m^3

Note that the mole fraction can be approximated by the (moles of salt/moles of water) as the number of moles of water is dominant, that is the mole fraction is close to $0.34 / 5.56 = 0.061$.

As the solution becomes more dilute, this approximation improves and generally for dilute solutions, the mole fraction of solute is a close approximation to the moles of solute / moles of solvent. In solid / liquid mixtures of all these methods can be used but in solid mixtures, the concentrations are normally expressed as simple weight fractions.

Example #6

In a textile mill, an evaporator concentrates a liquor containing solids of 6% by w/w (weight by weight) to produce an output containing 30% solids w/w. Calculate the evaporation of water per 100 kg of feed to the evaporator.

Solution

Inlet solid contents = 6%

Outlet solids contents = 30%

Feed = 100 kg

Solids content in kg in feed = $100 \times 0.06 = 6$ kg

Since mass in = mass out, the outlet solid content should be = 6 kg

Quantity of water evaporated = $[100 - (100/30) \times 6] = 80$ kg

Example #7: Air Composition

If air consists of 77% by weight of nitrogen and 23% by weight of oxygen calculate:

- 1) the mean molecular weight of air
- 2) the mole fraction of oxygen
- 3) the concentration of oxygen in $mole/m^3$ and kg/m^3 , if the total pressure is 1.5 atmospheres and the temperature is 25 °C

Solution

- 1) Taking the basis of 100 kg of air: it contains $77/28$ moles of N_2 and $23/32$ moles of O_2

Total number of moles = $2.75 + 0.72 = 3.47$ moles.

So mean molecular weight of air = $100 / 3.47 = 28.8$

Mean molecular weight of air = 28.8

2) The mole fraction of oxygen = $0.72 / (2.75 + 0.72) = 0.72 / 3.47 = 0.21$

Mole fraction of oxygen = 0.21

3) With gases, concentrations are primarily measured in weight concentrations per unit volume, or as partial pressures. These can be related through the gas laws. Using the gas law in the form:

$$pV = nRT$$

Where, p is the pressure, V the volume, n the number of moles, T the absolute temperature in K, and R the specific ideal-gas constant which is equal to $0.08206 \text{ m}^3\text{atm} / \text{mole K}$. The molar concentration of a gas is then $n / V = p/RT$ and the weight concentration is then nM/V where M is the molecular weight of the gas.

Thus for volume of 1 m^3 , temperature of $25 \text{ }^\circ\text{C} = 25 + 273 = 298 \text{ K}$, R is $0.08206 \text{ m}^3\text{atm} / \text{mole K}$ and pressure of 1.5 atm,

$$pV = nRT \text{ and so, } 1.5 \times 1 = n \times 0.08206 \times 298$$

$$n = 0.061 \text{ mole/m}^3$$

Weight of air = $n \times \text{mean molecular weight}$

= $0.061 \times 28.8 = 1.76 \text{ kg} / \text{m}^3$ and of this 23% is oxygen, so weight of oxygen = $0.23 \times 1.76 = 0.4 \text{ kg}$ in 1 m^3

Concentration of oxygen = 0.4kg/m^3 or $0.4 / 32 = 0.013 \text{ mole} / \text{m}^3$

When a gas is dissolved in a liquid, the mole fraction of the gas in the liquid can be determined by first calculating the number of moles of gas using the gas laws, treating the volume as the volume of the liquid, and then calculating the number of moles of liquid directly.

Example # 8: Gas composition

In the carbonation of a soft drink, the total quantity of carbon dioxide required is the equivalent of 3 volumes of gas to one volume of water at $0 \text{ }^\circ\text{C}$ and atmospheric pressure. Calculate (a) the mass fraction and (b) the mole fraction of the CO_2 in the drink, ignoring all components other than CO_2 and water.

Basis 1 m^3 of water = 1000 kg

Volume of carbon dioxide added = 3 m^3

From the gas equation, $pV = nRT$

$$1 \times 3 = n \times 0.08206 \times 273$$

$n = 0.134$ mole.

Molecular weight of carbon dioxide = 44

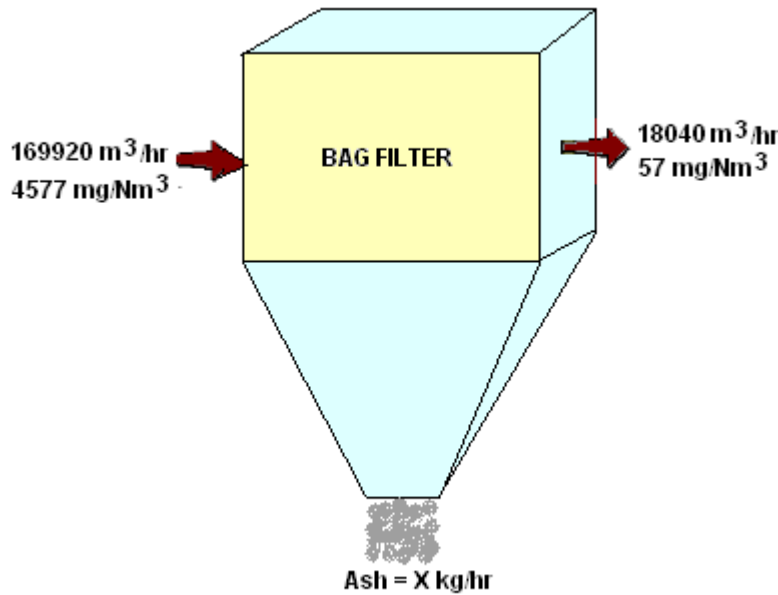
And so weight of carbon dioxide added = $0.134 \times 44 = 5.9$ kg

(a) Mass fraction of carbon dioxide in drink = $5.9 / (1000 + 5.9) = 5.9 \times 10^{-3}$

(b) Mole fraction of carbon dioxide in drink = $0.134 / (1000/18 + 0.134) = 2.41 \times 10^{-3}$

Example # 9: Air Pollution Mass Balance Calculation

A fabric filter (bag filter) is used to remove the dust from the inlet gas stream so that outlet gas stream meets the required emission standards in a chemical industry. During an air pollution monitoring study, the inlet gas stream to a bag filter is $169,920\text{m}^3/\text{hr}$ and the dust loading is 4577 mg/m^3 . The outlet gas stream from the bag filter is $185040\text{m}^3/\text{hr}$ and the dust loading is 57 mg/m^3 . What is the maximum quantity of ash that will have to be removed per hour from the bag filter hopper based on these test results?



CONSERVATION OF MATTER

Solution:

Based on dust balance,

Mass (in) = Mass (out)

Inlet gas stream dust = outlet gas stream dust + Hopper Ash

1) The inlet and outlet dust quantities in kg per hour

Inlet dust quantity = $169920\text{ (m}^3/\text{hr)} \times 4577\text{ (mg/m}^3) \times 1/1000000\text{ (kg/mg)} = 777.7\text{ kg/hr}$

Outlet dust quantity = $185040\text{ (m}^3/\text{hr)} \times 57\text{ (mg/m}^3) \times 1/1000000\text{ (kg/mg)} = 10.6\text{ kg/hr}$

2) Quantity of ash that will have to remove from the hopper per hour

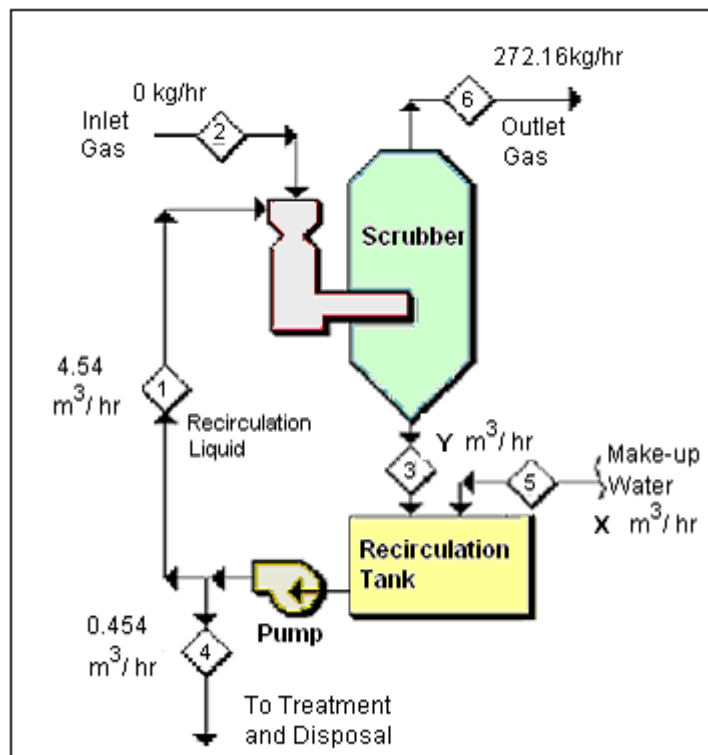
Hopper ash = Inlet gas dust quantity - Outlet gas dust quantity

$$= 777.7 \text{ kg/hr} - 10.6 \text{ kg/hr}$$

$$= 767.1 \text{ kg/hr}$$

Example # 10: Environmental Material Balance Analysis of a Process Operation

A scrubber is used to remove the fine material or dust from the inlet gas stream with a spray of liquid (typically water) so that outlet gas stream meets the required process or emission standards. The process flow details are depicted in figure below:



ENVIRONMENTAL FUME TREATMENT SCRUBBER MATERIAL FLOW DIAGRAM

How much water must be continually added to wet scrubber shown in figure above in order to keep the unit running? Each of the streams is identified by a number located in a diamond symbol. Stream 1 is the recirculation liquid flow stream back to the scrubber and it is $4.54 \text{ m}^3/\text{hr}$. The liquid being withdrawn for treatment and disposal (stream 4) is $0.454 \text{ kg m}^3/\text{hr}$. Assume that inlet gas stream (number 2) is completely dry and the outlet stream (number 6) has 272.16 kg/hr of moisture evaporated in the scrubber. The water being added to the scrubber is stream number 5.

Solution:

Objective – Solve for two unknown streams – 3 and 5

Method - Split the process into two parts, the one being scrubber and the second being the recirculation tank. Write the material balance equations for both the processes and solve the one which involve the fewest unknown variables.

The scrubber in this example has only one unknown i.e. stream 3 while the recirculation tank has two unknowns 3 and 5. Make sure before solving the problem convert all streams into identical units. In this case the stream 6 can be converted to m³/hr as

$$\begin{aligned}\text{Stream 6} &= 272.16 \text{ kg/hr} \times \text{m}^3/1000 \text{ kg} \\ &= 0.272 \text{ m}^3/\text{hr}\end{aligned}$$

Note, the conversion is approximate and this applies only to pure water.

Step -1: Material Balance around the Scrubber

Input Scrubber = Output Scrubber

Stream 1 + Stream 2 = Stream 3 + Stream 6

$$4.54 \text{ m}^3/\text{hr} + 0 = Y \text{ m}^3/\text{hr} + 0.272 \text{ m}^3/\text{hr}$$

$$Y = 4.27 \text{ m}^3/\text{hr} \text{ (Stream 3)}$$

Step 2: Material Balance around the Recirculation Tank

Input Tank = Output Tank

Stream 3 + Stream 5 = Stream 1 + Stream 4

$$4.27 \text{ m}^3/\text{hr} + X \text{ m}^3/\text{hr} = 4.54 \text{ m}^3/\text{hr} + 0.454 \text{ m}^3/\text{hr}$$

$$X = 4.994 \text{ m}^3/\text{hr} - 4.27 \text{ m}^3/\text{hr} = 0.724 \text{ m}^3/\text{hr} \text{ (Stream 5)}$$

Alternatively the makeup water (Stream 5) is the amount of water entering is equivalent to the amount of water leaving the process or

Stream 5 = Stream 4 + Stream 6

$$= 0.454 + 0.272$$

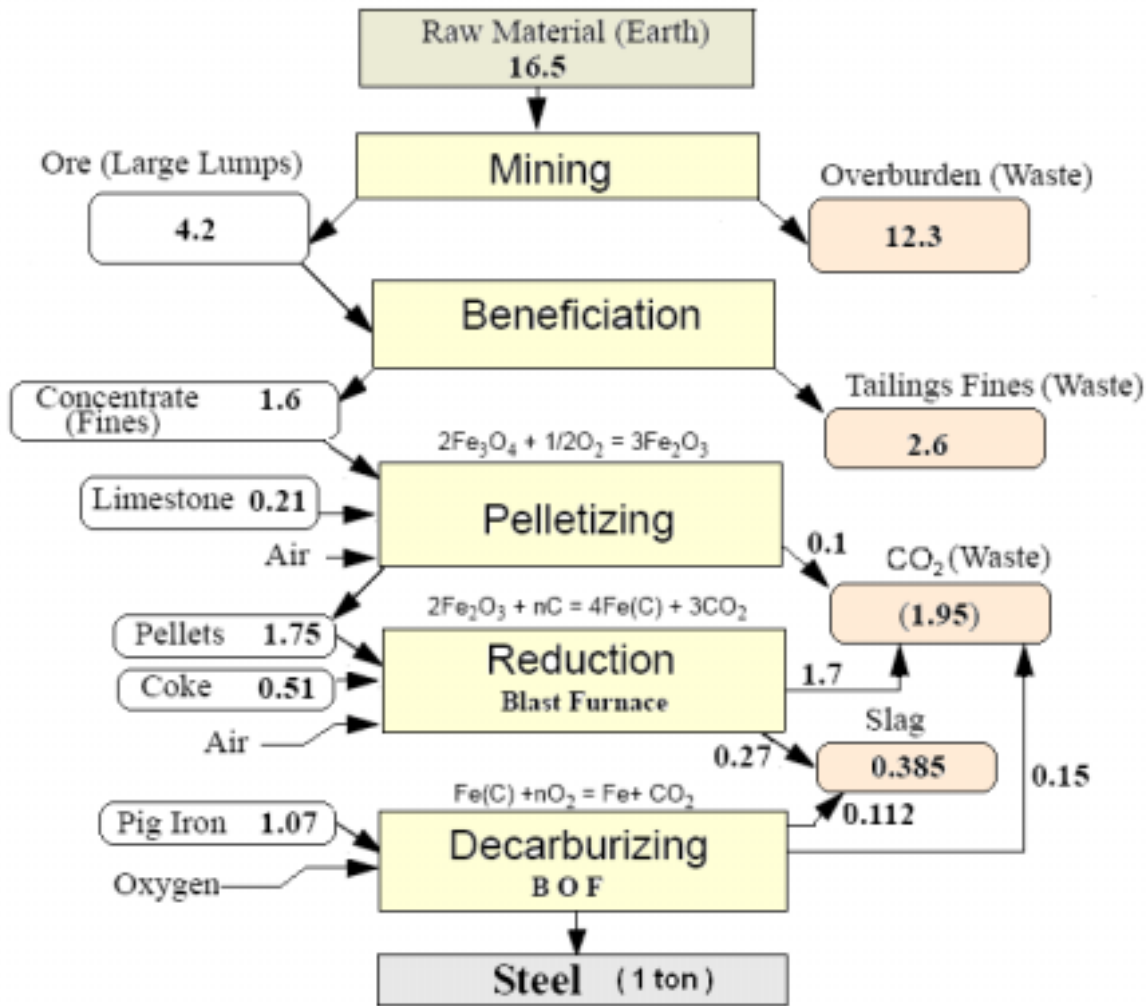
$$= 0.73 \text{ m}^3/\text{hr}$$

One of the key steps in solving example above was the flow diagram. This is absolutely necessary so that it is possible to conduct the material balances. A flow diagram is a valuable first step when solving a wide variety of problems and is very useful way to summarize what we know and what we need to know.

It helps visualize the solution. If the problem involves dimensional quantities (such as stream flow quantities), the dimensions should be included on the sketch. They serve as reminders of the need to convert the data into consistent units.

Material Flow in the Production of Steel

Steel is basically an alloy of iron (Fe) and carbon (C), with C present in amounts usually less than 0.8 wt. pct. A flow diagram for the production of steel, together with the physical and chemical changes accompanying the material are illustrated in figure below for each unit operation in the steel making process (Mining, Beneficiation, Pelletizing, Reduction in Blast Furnace, Decarburization in the Basic Oxygen Furnace (BOF)). The changes accompanying each unit operation require significant inputs of energy and/or other materials. Flow chart is constructed so that the weight in metric tons of materials needed to produce one metric ton of steel are indicated (numbers in the figure).



MATERIAL FLOW DIAGRAM FOR STEEL PRODUCTION

(All numbers in metric tons)

Let's now see the material balance for individual process streams.

MINING & CONCENTRATING

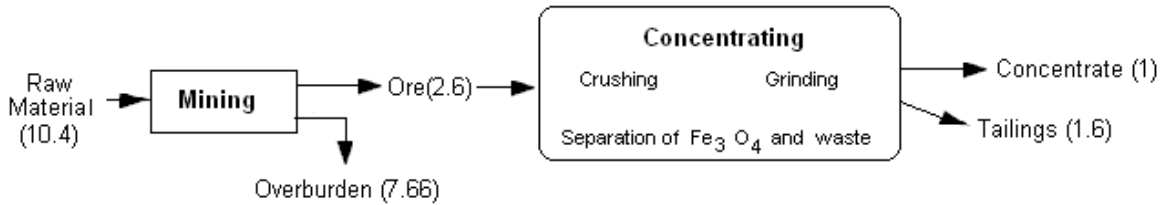
The mass balance shown below illustrates the mining and concentrating operation.

Material - IN [Composition, Wt%]

- ❖ Ore into concentrator [38.7 Fe₃O₄, 7.15 Fe₂O₃, 54.15 SiO₂]

Material - OUT [Composition, Wt%]

- ❖ Concentrated Ore [91.16 Fe₃O₄, 8.84 SiO₂]
- ❖ Tailings [6.0 Fe₃O₄, 11.6 Fe₂O₃, 82.4 SiO₂]



Simplified Mining and Beneficiation Mass Balances
(All numbers in metric tons)

An ideal ore would contain only the magnetite (Fe₃O₄), a result achievable only by spending a much larger amount of energy (and \$) in processing. A cost effective concentrate requires the presence of some of the impurity SiO₂. By the same token, the tailings (or waste product) will contain a significant amount of iron.

Example Calculation:

Considering the material balance (mass in /mass out) above, how much Fe in wt. pct. is disposed of in tailings ponds? This requires first determining the wt. fraction of Fe in Fe₂O₃ and Fe₃O₄.

$$\text{Wt. Fraction Fe (Fe}_2\text{O}_3) = \frac{2 \times \text{Atomic wt. Fe}}{2 \times \text{Atomic wt. Fe} + 3 \times \text{Atomic wt. O}} = \frac{2 \times 55.85}{2 \times 55.85 + 3 \times 16.01}$$

$$= \frac{111.7}{159.73} = 0.699$$

And

$$\text{Wt. Fraction Fe (Fe}_3\text{O}_4) = \frac{3 \times 55.85}{3 \times 55.85 + 4 \times 16.01} = \frac{167.55}{231.59} = 0.723$$

Using the concentrations given then in flow diagram above, wt. % Fe in Tailings = 6 * 723 + 11.6 * 699 = 4.338 + 8.108 = 12.446%.

Perhaps in many years the tailings pond will become an ore body. However, this is not economical at this time.

PELLETIZING (High Temperature Sintering)

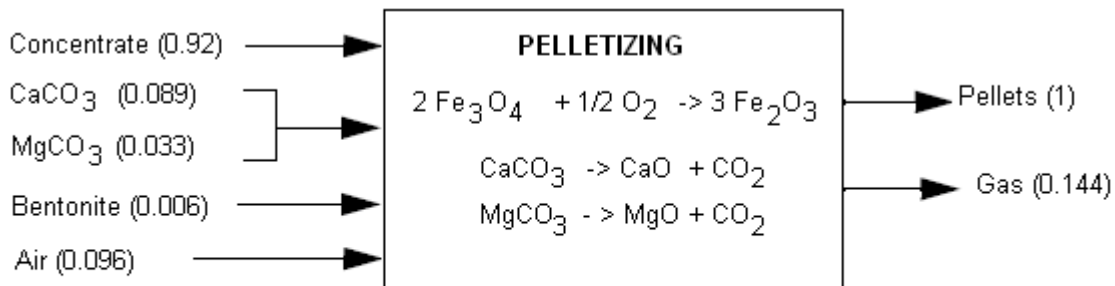
The mass balance shown in figure below illustrates the production of what is called a fluxed pellet. Instead of adding the flux to the blast furnace, it is incorporated into the pellets. The purpose of the flux is to produce slag in the blast furnace, a liquid layer formed on the surface of the pig iron which is effective in removing certain impurities. Flux materials are CaCO₃ and MgCO₃. The bentonite is added to aid in producing strong bonds during the heat of palletizing. The product is a pellet which will not crush during transportation or as a burden in the blast furnace.

Materials - IN [Composition, Wt %]

- ❖ Iron Ore Concentrate [91.2 Fe₂O₃, 8.8SiO₂]
- ❖ Flux [72 CaCO₃, 28 MgCO₃]
- ❖ Air [23.3 O₂, 76.7 N₂]
- ❖ Bentonite [(OH)4Al₄Si₈O₂₀]

Materials - OUT [Composition, Wt %]

- ❖ Pellets [87.3 Fe₂O₃, 5.8 SiO₂, 5CaO, 1.6 MgO, 0.3 Al₂O₃]
- ❖ Gas [CO₂, N₂]



SIMPLIFIED PELLETIZING MATERIALS BALANCE
(All numbers in metric tons)

Example Calculation:

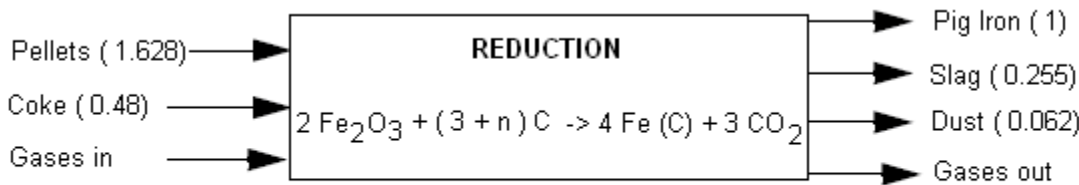
The limestone (CaCO₃) and dolomite (MgCO₃) are calcined (CO₂ is driven off) during pelletizing. Determine the weight of CO₂ generated during production of one ton of pellets. Atomic weights: Ca–40.08; Mg–24.32; O–16; C–12.01.

$$\text{Wt. CO}_2 = 178 \text{ lb} \left[\frac{\text{Atomic wt. CO}_2}{\text{Atomic wt. CaCO}_3} \right] + 67 \left[\frac{\text{Atomic wt. CO}_2}{\text{Atomic wt. MgCO}_3} \right]$$

$$= 178 \left[\frac{44.01}{100.09} \right] + 67 \left[\frac{44.01}{84.33} \right] = 112.96 \text{ lb}$$

REDUCTION – BLAST FURNACE

The blast furnace is a complex chemical reaction chamber in which hot gases together with a large amount of coke accomplish the reduction of Fe₂O₃ to hot metal containing a large quantity of carbon. Input gases include oxygen and coke oven gas, a complex mix of methane, hydrogen, carbon monoxide, and other volatiles. The countercurrent nature of the process results in significant quantities of iron going into dust as Fe₂O₃, a product which may be recovered and sintered to produce a size and shape compatible with recharging into the blast furnace.



SIMPLIFIED BLAST FURNACE MATERIAL BALANCE
(Production of one metric ton of Pig Iron)

Materials - IN [Composition, Wt %]

Pellets [87.3 Fe₂O₃, 5.8 SiO₂, 5CaO, 1.6MgO, 0.3 Al₂O₃]

Coke [90 C, 6 SiO₂, 3.5 Al₂O₃, 0.5CaO]

Gases in [O₂, H₂, CH₄, C₂H₆, CO, CO₂]

Materials - OUT [Composition, Wt %]

Pig Iron [95 Fe, 4.2 C, 0.8Si]

Slag [48.3 SiO₂, 33CaO, 8.4Al₂O₃, 10.3MgO]

Dust [Fe₂O₃]

Gases out [H₂O, CO, CO₂]

Example Calculation:

Given the compositions and amounts of materials shown going into the blast furnace, verify that the slag quantity of 0.255 tons per ton of pig iron and the composition shown are accurate.

Assuming the slag comes from the non-ferrous components of the pellets and the coke, then

$$\text{Wt. Slag} = 1.628 (.127) + 0.48 (0.1) = 0.207 + 0.048 = 0.255 \text{ metric ton}$$

Composition of slag:

$$\text{Wt. \%age SiO}_2 = 100 \times \frac{3256 (0.058) + 960 (0.06)}{510} = \frac{188.8 + 57.6}{510} \times 100 = 48.3$$

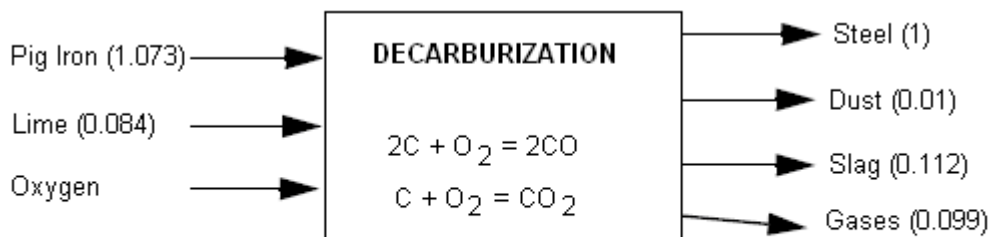
$$\text{Wt. \%age CaO} = 100 \times \frac{3256 (0.05) + 960 (0.005)}{510} = \frac{162.8 + 4.8}{510} \times 100 = 32.8$$

Etc.

DECARBURIZATION OF STEEL – Basic Oxygen Furnace

The Basic Oxygen Furnace (BOF) is a refining furnace whose primary function is to decarburize the pig iron from the blast furnace. In this process, the heat needed is supplied by the reaction of oxygen with the various constituents of the charge (mainly carbon in the pig iron). A typical BOF charge will have a mix of 70% hot metal (pig iron at 2400-2450°F) and 30% cold steel scrap, which when blown with oxygen will raise the temperature of the final steel product to ~3000°F.

Again, as in the blast furnace, lime is added to generate the necessary slag, which in this case will contain a significant quantity of iron, a quantity which is recovered using the BOF slag as a flux material in blast furnace burdens not using fluxed pellets. Figure below illustrates a simplified materials balance for a BOF operation in which no scrap steel is used. This is done only to simplify the calculations which follow, and is not meant to represent a typical BOF charge.



SIMPLIFIED (BOF) MATERIAL BALANCE
(Production of one metric ton of steel)

Material -IN [Composition, Wt %]

Pig Iron [95 Fe, 4.2 C, 0.8Si]

Lime [100CaO]

Oxygen [100O₂]

Material - OUT [Composition, Wt %]

Steel [99.3 Fe, 0.4 C, 0.3Si]

Slag [25 Fe₂O₃, 75CaO]

Dust [100Fe₂O₃]

Gases [90 CO, 10CO₂]

Exercise

In a drying process moisture is reduced from 60% to 30%. Initial weight of the material is 200 kg. Calculate the final weight of the product.

- a. 100
- b. 120
- c. 130
- d. 114.3

Do it yourself first.....

Scroll for answer below.....

Solution:

Initial mass = 200 kg

Initial moisture = 60%

Dry product = 200 – 200 * 0.6 = 80 kg

Let the mass of final product after drying is Y

Moisture in final product = 30% or 0.3Y

Applying mass balance equation:

Dry product + Moisture mass in final product = Mass of final product

$$80 + 0.3 Y = Y$$

Or

$$Y = 114.3 \text{ kg}$$

ENERGY BALANCES

Energy is the capacity to do work or to transfer heat.

The law of conservation of energy states that energy can neither be created nor destroyed. The total energy in the materials entering the processing plant, plus the energy added in the plant, must equal the total energy leaving the plant. This is a more complex concept than the conservation of mass, as energy can take various forms such as kinetic energy, potential energy, heat energy, chemical energy, electrical energy and so on. During processing, some of these forms of energy can be converted from one to another; say for instance mechanical energy in a fluid can be converted through friction into heat energy or chemical energy in food is converted by the human body into mechanical energy. It is the sum total of all these forms of energy that is conserved.

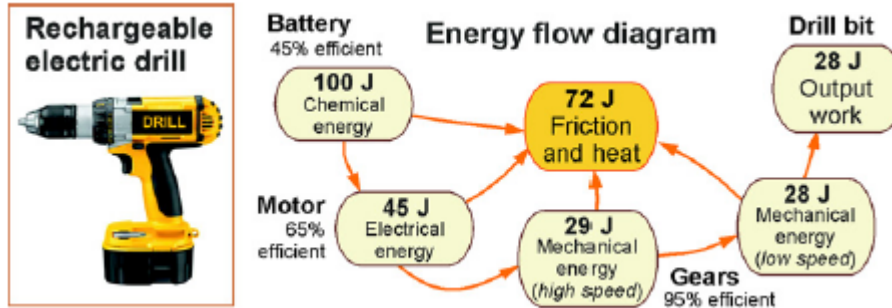
Example: Consider the pasteurizing process for milk, in which milk is pumped through a heat exchanger and is first heated and then cooled. The energy affecting the product is the heat energy in the milk. Heat energy is added to the milk by the pump and by the hot water passing through the heat exchanger. Cooling water then removes part of the heat energy and some of the heat energy is also lost to the surroundings. The heat energy leaving in the milk must equal the heat energy in the milk entering the pasteurizer plus or minus any heat added or taken away in the plant.

Heat energy leaving in milk = initial heat energy
+ heat energy added by pump
+ heat energy added in heating section
- heat energy taken out in cooling section
- heat energy lost to surroundings.

The law of conservation of energy can also apply to part of a process. For example, considering only the heating section of the heat exchanger in the pasteurizer, the heat lost by the hot water must be equal to the sum of the heat gained by the milk and the heat lost from the heat exchanger to its surroundings.

Energy flow diagrams

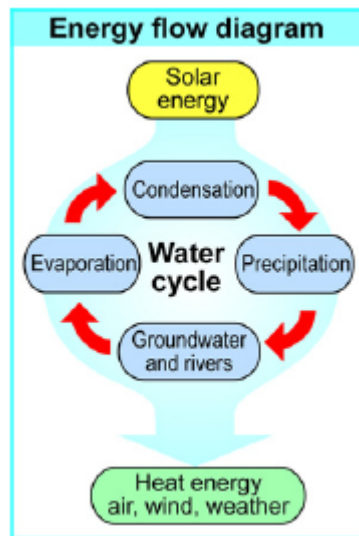
An energy flow diagram is a good way to show what happens to the energy in a system that is changing. To make an energy flow diagram, first write down the different forms that energy takes in the system. For example take a case of battery operated drill machine. Chemical energy, stored in the battery, is converted to electrical energy flowing through wires. The motor converts electrical energy to mechanical energy. The rotation of the motor is transferred to the drill bit by gears. The output work of the drill is force turning the drill bit and cutting wood. The energy balance for this application is illustrated below:



The diagram shows that input energy is 100J out of which the useful work is 28J and 72J is wasted energy due to friction and heat.

Steady state energy balance

Unlike mechanical systems, energy flow in natural systems tends to be in a steady state. Steady state means there is a balance between energy in and energy out so that the total energy remains the same.



For example, on earth, radiant energy from the sun is energy input. This energy is converted into many different forms through different processes. Much of the energy from the sun is absorbed by oceans and lakes and used to drive the water cycle. Some water evaporates into the air, carrying energy from the warm water into the atmosphere. The water vapor goes up into the atmosphere and cools, releasing its energy to the air. The cooled water condenses into droplets as precipitation, which falls back to the ground.

Eventually, the rainwater makes its way back to the ocean through rivers and groundwater and the cycle begins again. The water cycle moves energy from the oceans into the atmosphere and creates weather.

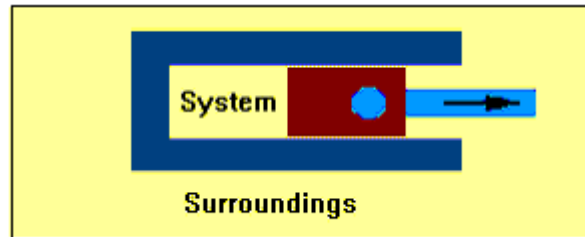
However, the average energy of the earth stays about the same because energy input is balanced by energy radiated back into space (energy output).

SYSTEM

A system is a collection of matter within defined boundaries. There are two types of system: closed system and open system.

Closed System

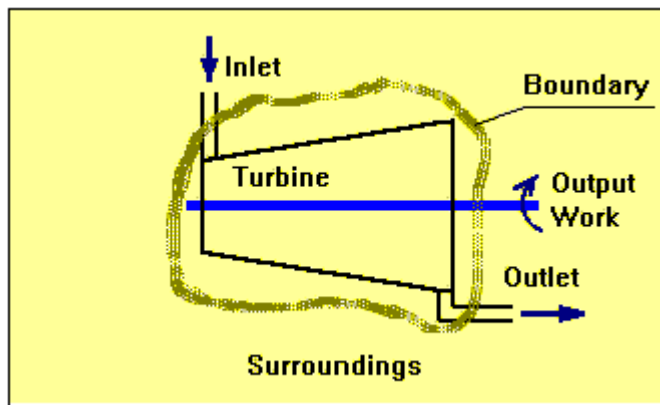
In closed systems, nothing leaves the system boundaries. A truly closed system would have to be completely insulated or completely isolated. As an example, consider the fluid in the cylinder of a reciprocating engine during the expansion stroke. The system boundaries are the cylinder walls and the piston crown. Notice that the boundaries move as the piston moves.



This system does not exist in nature; however, our solar system can be considered an essentially closed system.

Open System

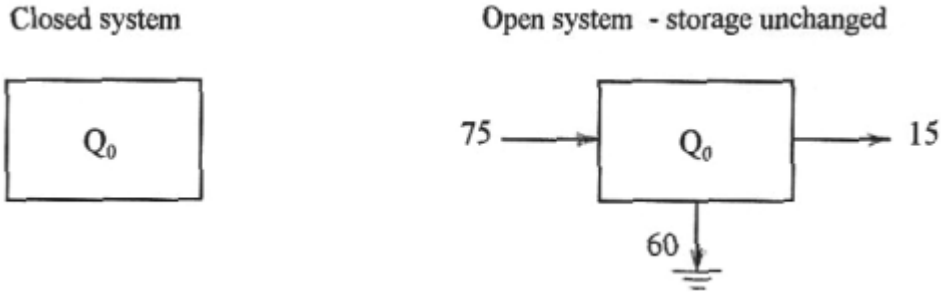
In open systems there is a mass transfer across the system's boundaries; most systems are open -- they interact with their environment. For instance the steam flow through a steam turbine at any instant may be defined as an open system with fixed boundaries.



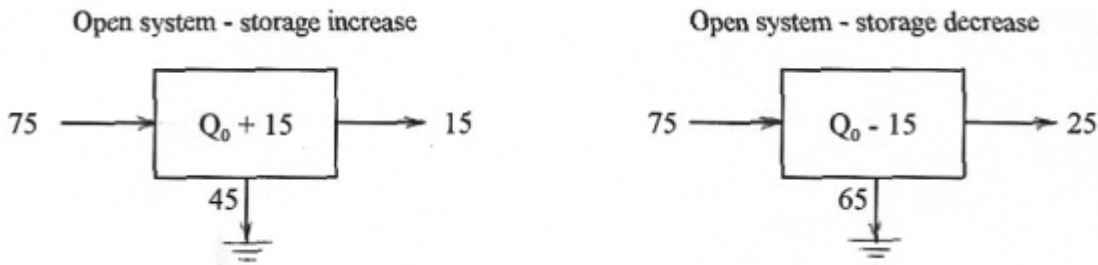
A candle imports oxygen and it exports carbon dioxide, carbon particles and heat. The more complicated example of a pond takes in air, water and sediment, while yielding insects, fish and water plants. Most human-designed organizations are open -- they have inputs and outputs.

Conservation of Energy:

Here is a fundamental principle of physics, known as the first law of thermodynamics -- *Energy can neither be created nor destroyed, it can only be transformed.* In a closed system or tank, the energy remains constant i.e. if the energy at the start is Q_0 then it remains at Q_0 . In an open system, if the storage does not change, the ingoing and outgoing energy must be equal.



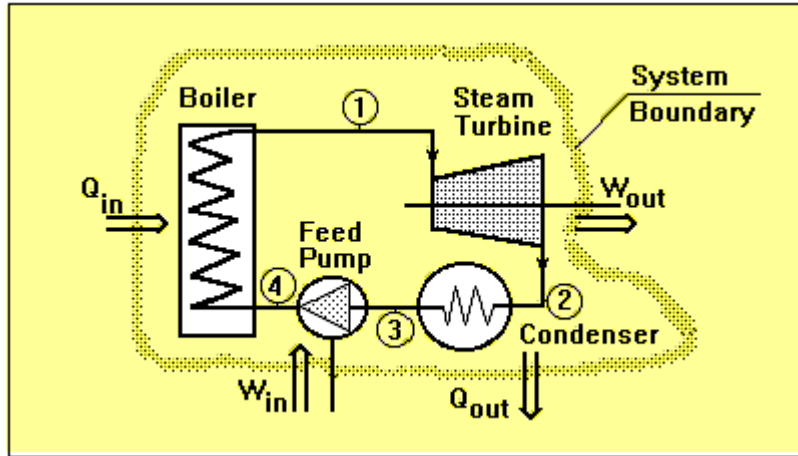
If the storage changes, this must be reflected in the energy balance and the energy input to a system might not balance the energy that goes out. For instance as in fig below, the system on the left, the input is 75 units of energy but only 60 units go out. Since the first law requires that the energy be conserved, system had to *gain* 15 units of energy. In the right system the input is short 15 units of energy so we can infer that the system must have lost 15 units.



The sinks are depositories of leakage or rejected energy. It is usually low-grade heat, as in respiration, an engine exhaust, or a non-insulated hot water heater. The outputs represent useful work.

Example

Figure below illustrates power cycle schematic. In this input heat energy (Q_{IN}) resulting from combustion of fuel is transferred to water in a steam generator (boiler). The fluid feed water is pumped using input energy W_{IN} . The steam is used to drive the turbine and perform useful work W_{OUT} and the steam is condensed in condenser giving its heat energy Q_{OUT} . The working fluid is feed water which changes its state from water to steam and back to condensate.



Applying the law of conservation of energy, if a system undergoes a process by heat and work transfer, then the net heat supplied, Q , plus the net work input, W , is equal to the change of intrinsic energy of the working fluid, i.e.

$$\Delta U = U_2 - U_1 = Q + W$$

Where

U_1 and U_2 are intrinsic energy of the system at initial and final states, respectively. Applying this general principle to a thermodynamic cycle, when the system undergoes a complete cycle, i.e. $U_1 = U_2$, results in:

$$\Sigma Q + \Sigma W = 0$$

Where:

ΣQ = The algebraic sum of the heat supplied to (+) or rejected from (-) the system.

ΣW = The algebraic sum of the work done by surroundings on the system (+) or by the system on surroundings (-).

Applying the rule to the power plant gives:

$$\Sigma Q = Q_{IN} - Q_{OUT}$$

$$\Sigma W = W_{IN} - W_{OUT}$$

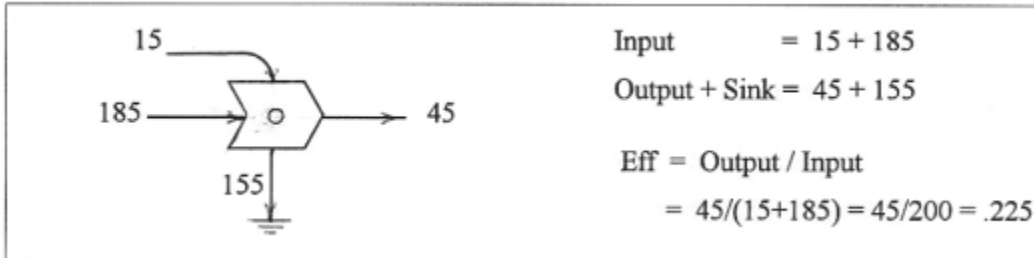
$$Q_{IN} + W_{IN} - Q_{OUT} - W_{OUT} = 0$$

where,

- Q_{IN} = Heat supplied to the system through boiler
- W_{IN} = Feed-pump work
- Q_{OUT} = Heat rejected from the system by condenser
- W_{OUT} = Turbine work

Efficiency

The (thermodynamic) efficiency of a process is the ratio of useful output to input and is always less than 100%. Consider an energy balance below. There is no internal storage, so the sum of the inputs must equal the outgoing energy. The efficiency for the below process is 22.5% and the energy lost in the system is $100 - 22.5 = 77.5\%$.



How can energy be lost in a system?

As discussed before, energy can manifest itself in many forms such as heat, kinetic energy, chemical energy, potential energy but because of inter-conversions, it is not always easy to isolate separate constituents of energy balances. Saying energy is “lost” really means it changes into a form you are not counting. Most often the “uncounted” energy is work done against friction. This work changes other forms of energy into heat and wear. If you could measure every form of energy, you would find that the tires of the car and the track became a little warmer. Some rubber was worn off the tires and some wood was worn off the track. Wear means grinding away molecules from surfaces. This means breaking bonds between molecules, which takes energy. If you could add it all up you would find that all the energy at the start is still there at the end, just in different forms. This loss of usable energy is due to many causes –

- In mechanical systems it is friction
- In electrical systems it is resistance
- In fluid systems it is turbulence, viscosity or mixing
- In communication systems it is noise

We are seldom concerned with internal energies in manufacturing industry and for all practical approaches to energy conservation, the application of energy balances tend to focus on “heat balances”. When unfamiliar with the relative magnitudes of the various forms of energy (oil, gas, coal, steam, chilled water or electricity) entering and exiting a particular processing situation, it is best to put them all down and convert them to equivalent “Heat Energy”.

Measure of Heat (Thermal) Energy

One measure of heat energy is the BTU (British thermal unit). A BTU is the energy that goes into heating one pound of water one degree Fahrenheit. Amazingly, this is the equivalent of 778 ft-lbs of work. In other words, the thermal energy that goes into raising the temperature of an 11 ounce mug of coffee one degree

(°F) is approximately equivalent to the work of lifting 55 pounds up one flight of stairs. (A pint or 16 fluid ounces of water weighs about one pound. Rule of thumb -- *A pint's a pound the world around.*)

There are a seemingly endless number of energy units and the table below provides some common energy units and conversions.

Some Units of Energy & Conversion Factors

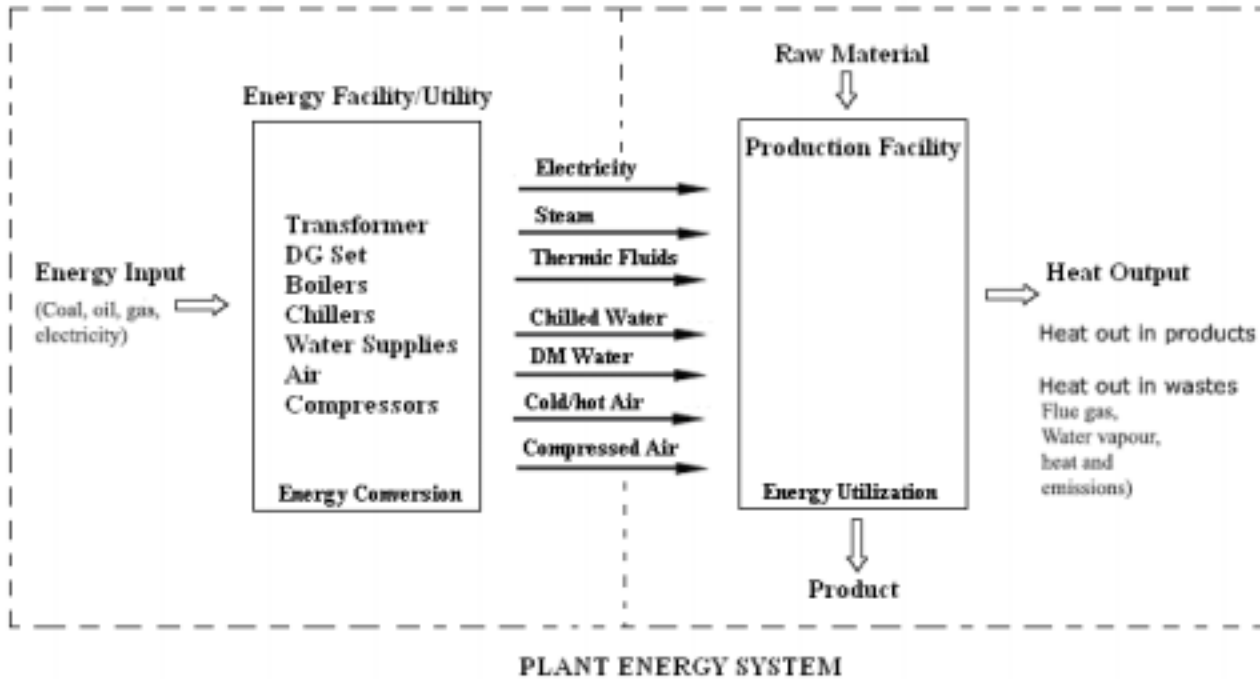
			BTU	Cal	ft-lb	joule	kW-hr
BTU	British Thermal Unit -- One BTU can raise the temperature of one pound of water one degree Fahrenheit (°F)	=	1	0.252	778	1055	-
Cal	Large or kilogram calorie -- One Cal can raise the temperature of one kilogram of water	=	3.97	1	3088	4186	-
ft-lb	The energy exerted by a force of one pound moving one foot	=	-	-	1	1.36	-
joule	The energy exerted by a force of one Newton moving one meter 1 joule/sec = watt	=	-	-	0.737	1	-
kW-hr	The energy it takes to run a 1000 watt appliance or light for one hour	=	3414	860	2.66*10 ⁶	3.60*10 ⁶	1

Facility as an Energy System - Heat Balances

The energy or heat balance is the most basic tool of the energy conservation engineer. The measurement and calculation of a detailed heat balance provides the detailed break down of the total losses and indicates to what extent the process can be optimized.

In a production facility, the primary energy in form of coal, oil, gas and electricity* enters the facility and is converted to more usable secondary energy such as steam, compressed air, chilled water. The outgoing

energy is usually in the form of heat and motion. (* electricity is also secondary form of energy as it is generated by thermal heat energy or hydro generation)



The typical relationship between energy use and production can be as illustrated in diagram 1(a). The energy consumption 'E' has two components namely:

- ❖ mP = energy related to production (P)
- ❖ e = energy not related to production

Where m is specific energy consumption i.e. energy used per unit of production; therefore total energy related to P units of production is mP .

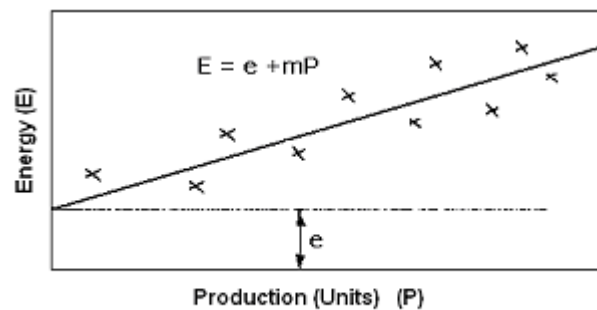


Diagram 1a

The diagram reveals a great deal about the efficiency of the process and the proportion of energy used irrespective of production rates. Ideally, if there were no service utilities (lighting, office air conditioning etc)

or losses, the energy used would be directly related to production as shown in diagram (1b) where $E = mP$ and $e = 0$.

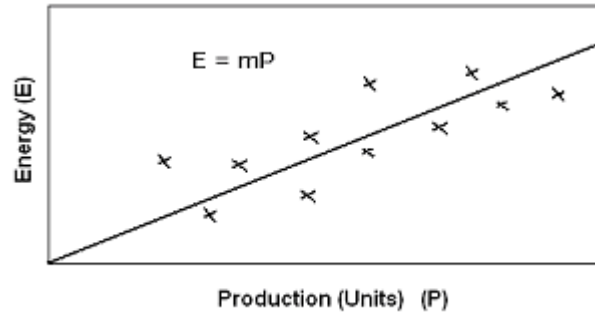


Diagram 1b

At the other extreme, a very inefficient process may use the same energy whether production is being carried out or not. The graph in this case would be as shown in diagram 1(c) and there should be major savings achievable by a change in existing equipment/working practices or even a complete redesign of the process. The exception would be where very low plant capacity utilization occurs. The non-production related energy consumption becomes disproportionately large simulating the case for diagram 2c.

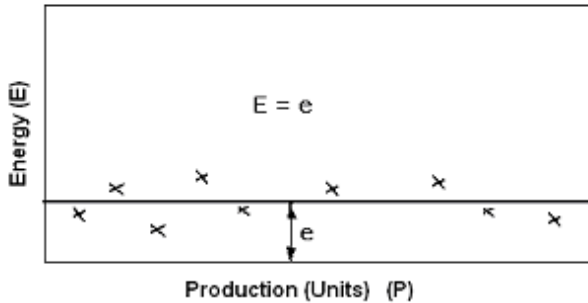


Diagram 1c

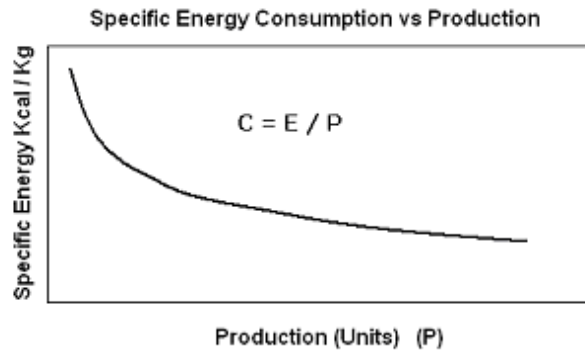


Diagram 2c

Specific Energy Consumption

Specific energy consumption (C) is defined as the ratio of energy used (E) to production (P), expressed as the energy requirement per unit of production. The units used will vary dependent on the fuel used for example it takes X Kcal of heat to produce 1kg of paper. This can also be expressed as Y Liter of fuel per kg of paper.

For an ideal process as illustrated in diagram 1(b)

$$C = E / P = m$$

This will only true where there are no service utilities and no losses.

In practical situation, as in diagram 1(a), the specific energy consumption is

$$C = E / P = (e + mP) / P = (e / P) + m$$

At high production rates (e/P) becomes very small and approaches the ideal case.

As mentioned previously, in the case where production rates are very low (e) becomes significant and (C) will become high. Diagram 2c shows how the (C) value varies with production in a typical practical case. All points below the 'performance curve' represent improved efficiency in energy usage. As part of any overall energy conservation program, each facility must try to bring this performance curve lower towards optimum criteria.

Heat Balance of Boiler

A heat balance is an attempt to balance the total energy entering a system against that leaving the system in different forms. The figure below illustrates the heat balance and different losses occurring while generating steam.

Evaluation of Thermal Efficiency

Direct Method: The direct method requires an ability to measure all inputs to the boiler and all discharges from it:

- Inputs – Fuel, energy in feed water, hot air for combustion
- Outputs – Steam, steam pressure, steam temperature, dryness fraction

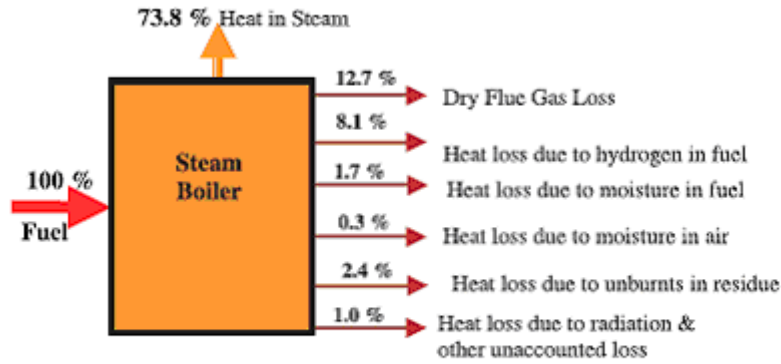
The evaluation is then simple:

Efficiency = Heat in steam above feed water temperature / Heat from fuel above entry temperature

It is difficult to accurately measure the dryness fraction of steam and only few sites will have accurate calibrated fuel and steam metering equipment thus; this method can only rarely be applied.

Indirect or losses method: This method evaluates the different losses in the system and the thermal efficiency is given by

$$\text{Thermal Efficiency (\%)} = 100 - (\text{Sum of all losses})$$



BOILER HEAT BALANCE

Analyzing the above losses, the major controllable heat losses are dry flue gas loss and heat loss due to unburnts in residue. These are the target areas for improvement:

Heat lost in dry flue gases- In practice this loss can vary from 8 to 35%. The main influencing factors are the exit flue gas temperature and the degree of excess air present. To minimize this loss for coal fired appliances better combustion is required including better fuel preparation and better stroking practices, improved control of combustion air both under-grate and over grate draughts. The same factors apply to oil making sure that fuel preparation is correct (uncontaminated and at the right temperature), burners are undamaged and properly maintained; combustion air (both primary and secondary) is introduced at the right rate and with the correct degree of turbulence. Other important considerations are the boiler loading, units operating should match the demand and operate at least at 60% of rated output; efficient waste recovery, this primarily refer to the use of three instead of two pass boilers and the inclusion of economizers, pre-heaters and minimizing cold air infiltration into the boiler at any point.

Heat loss to CO – By controlling the amount of dark smoke produced the level of CO can be kept to a practical minimum. The influencing factors are insufficient combustion air or inadequate fuel-air mixing or ingress of cold air ‘freezing’ the combustion reaction. In general the heat loss measured as the non-conversion of carbon into carbon dioxide is small but the factor which greatly affects the boiler’s performance is the rapid fouling of heat transfer surfaces that occurs.

Heat loss due to unburnts in residue - The loss generally varies between 2 to 4% and for coal appliances; the loss generally varies between 5 and 15%. This is clear indication of starvation of combustion air either because of poor distribution under the grate or poor stoking practices with uneven bed thickness or too thick a fire bed.

GENERATING ENERGY EFFICIENCY OPTIONS

The increasing cost of energy has caused the industries to examine means of reducing energy consumption in processing. Energy heat balances are used in the examination of the various stages of a process, over

the whole process and even extending over the total production system from the raw material to the finished product. Analyzing the waste streams, the following energy efficiency options may be implemented:

Boiler System

- Maximize combustion efficiency by adopting of combustion controls. Regulating the quantity of fuel and air flow can result in 0.25% increase in efficiency for each 1% decrease in excess O₂ depending on the stack gas temperature.
- Maximizing heat recovery between systems (e.g. using pre-heaters and economizer to recover heat and heat the combustion air and feed water respectively). 2.5% increase in efficiency is possible for each 100°F decrease in stack temperature.
- Minimizing blowdown and recovering heat from blowdown water to feed water can result in 1 to 3% efficiency improvement.
- Utilizing energy efficient burners with flame shape and heat release rate compatible with furnace characteristics.

Steam System

Search for situations where use of high pressure steam can be switched over feasibly to lower pressure steam. It is advantageous to use the lowest pressure for heat transfer and when high temperatures are not required. This is particularly true when the lower pressure steam is being supplied from extraction or back-pressure turbines or a low pressure boiler separate from the high pressure boiler. Of course, lowering pressure by a pressure reducing valve offers no savings in energy. Improving traps to eliminate steam leaks can result in significant energy savings.

Increase Condensate Return to Boilers

Loss of condensate is a waste of heat and of valuable high purity water. Identify all sources of condensate and evaluate economic feasibility of installing pumps and insulated piping to return condensate to the boiler feed water tank. If the condensate is contaminated, evaluate possible cleanup.

Improving condensate return reduces the hot requirements thus increasing the efficiency of operation 5 to 10% depending on the quantity of condensate returned.

Consider Cogeneration

Consider generating the steam at the highest steam pressure and dropping the pressure through a steam turbine. This will allow the highly efficient generation of electrical power for use in the process.

Electric Motors and Equipment

Electric motors and equipment, such as centrifugal pumps, operate with best efficiency at rated load. If they are operating at reduced load, efficiency suffers. For pumps and fans consider reducing speed (RPM),

impeller trimming if the flow rates and pressure are excessive. Consider variable speed drives or modifying pulley diameters for belt drives.

Insulation

Check potential energy losses for equipment operating at high temperatures. Make a determination if energy losses can be reduced economically by specifying better thermal insulation. Use ceramic fiber lining for better thermal insulation.

EXAMPLES

A few examples for energy generation, distribution and utilization are illustrated below:

Example #1: Combustion Analysis

A boiler uses coal as primary fuel for raising steam. The manufacturer recommends air-fuel ratio of 1:12 for optimum combustion i.e. 12 kg of air is required per kg of coal. Calorific value of coal is 4200 kCal /kg with ash content of 22%. What is the quantity (in kg) flue gas generated by burning 5 kg coal?

Flue gas quantity (per kg of coal) = combustion air + quantity of fuel- ash

$$= 12 + 1 - 0.22 = 12.78 \text{ kg}$$

Quantity of flue gas by burning 5 kg of coal: $5 \times 12.78 = 63.9\text{Kg}$

Example #2: Fuel Analysis

A sample of coal from the mine is found to contain 67.2% carbon and 22.3% ash. The refuse obtained at the end of combustion is analyzed to contain 7.1% carbon and the rest is ash. Compute the % of the original carbon unburnt in the refuse.

Data:

Coal – 67.2% carbon

Ash- 22.3%

Refuse – 7.1%

Carbon Ash – 92.9%

Solution

Basis: 100 kg of coal

Ash remains the same in refuse and coal

Mass of carbon in coal = 67.2 kg

Mass of ash in coal = 22.3 kg

Mass of ash in refuse = 22.3 kg

Mass of refuse = $100/92.9 \times 22.3 = 24 \text{ kg}$

Quantity of carbon in refuse = $7.1/100 \times 24 = 1.704 \text{ kg}$

% of original carbon remaining unburnt in the refuse = $1.704/67.2 \times 100 = 2.53\%$

Example #3: Heat Exchanger

A shell and tube heat exchanger is cooled with stream of demineralized water. Evaluate the total heat rejected to cooling water (kcal/hr), if the water flow rate is $200\text{m}^3/\text{hr}$ and the temperature rise is 7°C .

Solution

Heat rejected to the cooling water (Q) = $m \times C_p \times \Delta T$

Where

$m = \text{mass of water (kg /hr)} = 200 \times 1000 = 200000\text{kg/hr}$

$C_p = \text{Specific heat of water} = 1 \text{ kcal/kg } ^\circ\text{C}$

$\Delta T = \text{Temperature rise} = 7^\circ\text{C}$

Therefore

$Q = 200000 \times 1 \times 7 = 1400000 \text{ Kcal/hr}$

Example #4: Furnace

A furnace shell has to be cooled from 90°C to 55°C . The mass of the furnace shell is 2 tonnes; the specific heat of furnace shell is $0.2 \text{ kCal/kg } ^\circ\text{C}$. Water is available at 28°C . The maximum allowed increase in water temperature is 5°C . Calculate the quantity of water required to cool the furnace. Neglect heat loss.

Solution

Energy Stream #1

Mass of furnace shell (m) = 2000 kg

Specific heat (Cp) = $0.2 \text{ kCal/kg } ^\circ\text{C}$

Initial furnace temperature (T1) = 90°C

Desired furnace shell temperature (T2) = 55°C

Total heat that has to be removed from the furnace = $m \times C_p \times (T1 - T2) = 2000 \times 0.2 \times (90 - 55) = 14000 \text{ kCal}$

Energy Stream #2

Quantity of water required = X kg

Specific heat of water = $1 \text{ kCal/kg } ^\circ\text{C}$

Inlet cooling water temperature (T3) = 28°C

Maximum cooling water outlet temperature (T4) = 33°C

Heat removed by water = $X \cdot 1 \cdot (33 - 28) = 5X$ Kcal

For energy balance: Energy Stream #1 = Energy Stream #2

Or quantity of water required (X) = $14000/5 = 2800$ kg

Example #5: Boiler Blowdown

A boiler is fed with soft water containing 120 mg/l dissolved solids. As per IS standards the maximum dissolved solids in the boiler should not exceed 3500 mg/l for boilers, operating up to 2 MPa. In order to maintain the specified level, a continuous blow down system is adopted. Find the percentage of feed water which will be blown down.

Solution

Basis 1 kg of feed water

Let blow down quantity = X kg

Dissolved solids in blow down = 3500 mg/l

Per mass balance equation:

$$X \cdot 3500 = 120 \cdot 1$$

$$X = 0.0343 \text{ kg or}$$

$$\% \text{ blow down} = 0.0343/1 \times 100 = 3.43\%$$

Example #6: Dryer heat balance

A textile dryer is found to consume 4 m³/hr of natural gas with a calorific value of 800 kJ /mole. If the throughput of the dryer is 60 kg of wet cloth per hour, drying it from 55% moisture to 10% moisture, estimate the overall thermal efficiency of the dryer taking into account the latent heat of evaporation only.

Solution

- 1) Initial moisture in wet cloth = $60 \times 0.55 = 33$ kg moisture
- 2) Bone dry cloth = $60 \times (1 - 0.55) = 27$ kg bone dry cloth
- 3) Final product moisture content 10% = $27/9 = 3$ kg
- 4) So moisture removed /hr = $33 - 3 = 30$ kg/hr
- 5) Latent heat of evaporation = 2257 kJ/kg
- 6) Heat necessary to supply = $30 \times 2257 = 6.8 \times 10^4$ kJ/hr

- 7) Assuming the natural gas to be at standard temperature and pressure at which 1 mole occupies
- 8) 22.4 liters
- 9) Rate of flow of natural gas = $4 \text{ m}^3/\text{hr} = (4 \times 1000)/22.4 = 179 \text{ moles/hr}$
- 10) Heat available from combustion = $179 \times 800 = 14.3 \times 10^4 \text{ kJ/hr}$
- 11) Approximate thermal efficiency of dryer = heat needed / heat used = $6.8 \times 10^4 / 14.3 \times 10^4 = 48\%$

To evaluate this efficiency more completely it would be necessary to take into account the sensible heat of the dry cloth and the moisture, and the changes in temperature and humidity of the combustion air, which would be combined with the natural gas. However, as the latent heat of evaporation is the dominant term, the above calculation gives a quick estimate and shows how a simple energy balance can give useful information.

Example #7: Autoclave heat balance in canning

An autoclave contains 1000 cans of pea soup. It is heated to an overall temperature of 100 °C. If the cans are to be cooled to 40 °C before leaving the autoclave, how much cooling water is required, if it enters at 15 °C and leaves at 35 °C? Note the additional information below:

The specific heats of the pea soup and the can metal are respectively 4.1 kJ/ kg °C and 0.50 kJ/ kg °C.

The weight of each can is 60g and it contains 0.45 kg of pea soup.

Assume that the heat content of the autoclave walls above 40 °C is 16000 kJ and that there is no heat loss through the walls.

Solution

Let w = the weight of cooling water required; and the datum temperature be 40°C, the temperature of the cans leaving the autoclave.

Heat entering

- 1) Heat in cans = weight of cans x specific heat x temperature above datum
= $1000 \times 0.06 \times 0.50 \times (100-40) \text{ kJ} = 1800 \text{ kJ}$
- 2) Heat in can contents = weight pea soup x specific heat x temperature above datum
= $1000 \times 0.45 \times 4.1 \times (100 - 40) = 110700 \text{ kJ}$
- 3) Heat content in the autoclave = 16000 kJ
- 4) Heat in water = weight of water x specific heat x temperature above datum
= $w \times 4.186 \times (15-40) = -104.6 w \text{ kJ.}$
- 5) Total Heat Entering = $1800 + 110700 + 16000 - 104 w = 128500 - 104.6 w$

Heat leaving

- 1) Heat in cans = $1000 \times 0.06 \times 0.50 \times (40-40)$ (cans leave at datum temperature) = 0
- 2) Heat in can contents = $1000 \times 0.45 \times 4.1 \times (40-40) = 0$
- 3) Heat leaving the autoclave = 0
- 4) Heat in water = $w \times 4.186 \times (35-40) = -20.9 w$
- 5) Total heat leaving = $-20.9 w$

Heat – Energy balance of cooling process; 40°C as datum line

Total heat entering = Total heat leaving

$127800 - 104.6 w = -20.9 w$ or

$w = 1527 \text{ kg}$

Therefore amount of cooling water required = 1527 kg

Example #8- Refrigeration load

It is desired to freeze 10,000 loaves of bread each weighing 0.75 kg from an initial room temperature of 18°C to a final temperature of -18°C. The bread-freezing operation is to be carried out in an air-blast freezing tunnel. It is found that the fan motors are rated at a total of 80 horsepower and measurements suggest that they are operating at around 90% of their rating, under which conditions their manufacturer's data claims a motor efficiency of 86%. If 1 ton of refrigeration is 3.52 kW, estimate the maximum refrigeration load imposed by this freezing installation assuming (a) that fans and motors are all within the freezing tunnel insulation and (b) the fans but not their motors are in the tunnel. The heat-loss rate from the tunnel to the ambient air has been found to be 6.3 kW.

Extraction rate from freezing bread (maximum) = 104 kW

Fan rated horsepower = 80

Now 0.746 kW = 1 horsepower and the motor is operating at 90% of rating,

And so (fan + motor) power = $(80 \times 0.9) \times 0.746 = 53.7 \text{ kW}$

(a) With motors + fans in tunnel

Heat load from fans + motors = 53.7 kW

Heat load from ambient = 6.3 kW

Total heat load = $(104 + 53.7 + 6.3) \text{ kW} = 164 \text{ kW}$

= 46 tons of refrigeration

(b) With motors outside, the motor inefficiency = $(1 - 0.86)$ does not impose a load on the refrigeration

Total heat load = $(104 + [0.86 \times 53.7] + 6.3)$
= 156 kW
= 44.5 tons of refrigeration

In practice, material and energy balances are often combined as the same stoichiometric information is needed for both.

Example #9 – Evaporation Rate

Production rate from a paper machine is 340 tonnes per day (TPD). Inlet and outlet dryness to paper machine is 40% and 95% respectively. Evaporated moisture temperature is 80 °C. To evaporate moisture, the steam is supplied at 3.5 kg/cm². Latent heat of steam at 3.5 kg/cm² is 513kCal/kg.

Assume 24 hours/day operation a) Estimate the quantity of moisture to be evaporated b) Input steam quantity required for evaporation (per hour). Consider enthalpy of evaporated moisture as 632kcal/kg.

Solution

Production rate from a paper machine: 340 TPD or 14.16 TPH (tonnes per hour)

Inlet dryness to paper machine: 40%

Outlet dryness from paper machine: 95%

Estimation of moisture to be evaporated

Paper weight in final product: $14.16 \times 0.95 = 13.45$ TPH

Weight of moisture before dryer: $[(100-40) / 40] = 20.175$ TPH

Weight of moisture after dryer: $[(100- 95) / 95] = 0.707$ TPH

Evaporated moisture quantity: $20.175 - 0.707 = 19.468$ TPH

Input steam quantity required for evaporation

Evaporated moisture temperature: 80 °C

Enthalpy of evaporated moisture: 632kCal/kg

Heat available in moisture (sensible & latent): $632 \times 19468 = 12303776$ kCal/h

For evaporation minimum equivalent heat available should be supplied from steam

Latent Heat available in supply steam (at 3.5 kg/cm² (a)) = 513 kCal/kg

Quantity of steam required: 23984 kg or 23.98 MT/hour

Energy Analysis and the Sankey Diagram

The basic data needed for an energy analysis is an energy balance of each process section. The objective is to define in detail the energy input, energy utilized, and the energy dissipated or wasted. This is best represented by a Sankey diagram.

The Sankey diagram is very useful tool to represent an entire input and output energy flow in any energy equipment or system such as boiler generation, fired heaters, furnaces after carrying out energy balance calculation. Usually the flows are represented by arrows. The width of the arrows is proportional to the size of the actual flow. Better than numbers, tables or descriptions, this diagram represents visually various outputs (benefits) and losses so that energy managers can focus on finding improvements in a prioritized manner.

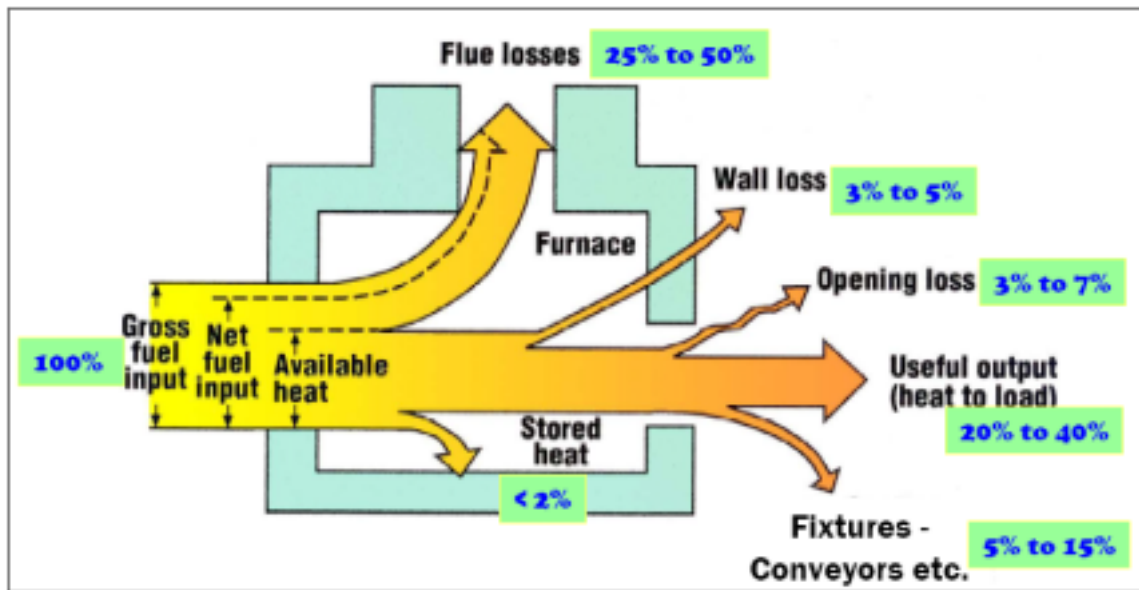


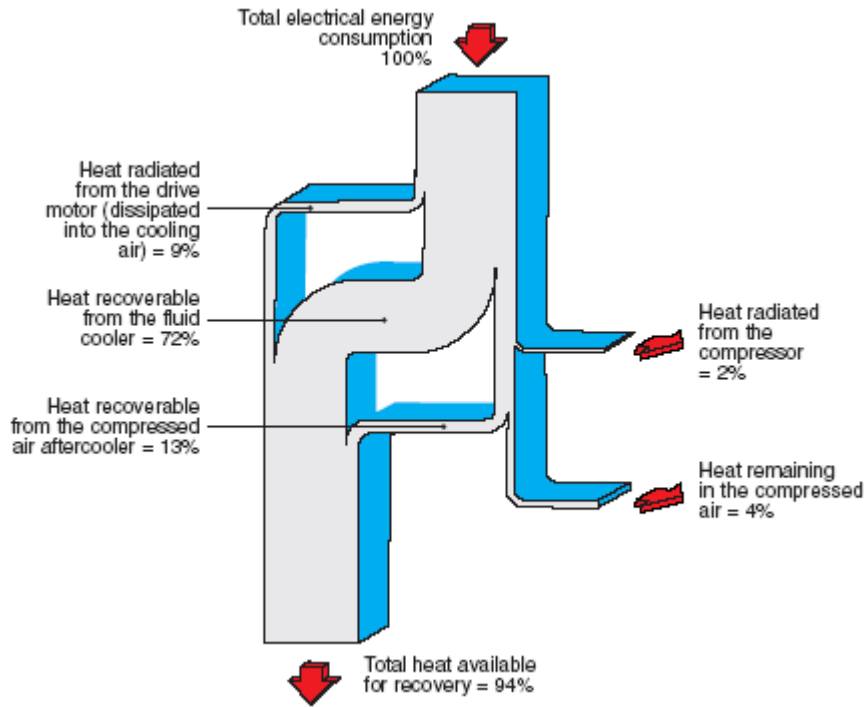
Figure above shows a Sankey diagram for a reheating furnace, which shows clearly that exhaust flue gas losses are a key area for priority attention. Since the furnaces operate at high temperatures, the exhaust gases leave at high temperatures resulting in poor efficiency. Hence a heat recovery device such as air preheater has to be necessarily part of the system. The lower the exhaust temperature, higher is the furnace efficiency.

Having identified the individual energy uses and wastes, the engineer can determine if these energy wastes could be recovered economically.

Example- Compressor Heat Balance

Almost all the electrical energy consumed by a compressor is changed into heat. On screw compressors, approximately 94% of this heat is given up to the cooling system, approximately 4% remains in the compressed air and approximately 2% is radiated from the compressor into the immediate surroundings. Further the 94% of the overall electrical energy loss consists of heat dissipated in the fluid cooler (72%), the

after-cooler (13%) and heat radiated from the drive motor (9%). Refer to the heat flow Sankey diagram below:



Because of the high cost of electricity today, it is important to minimize this loss or to recover it efficiently by transferring the heat into a medium and then transporting it to where the heat can be utilized. If water is to be heated, the oil in the fluid cooler is chosen as the transfer medium so that only 72% of the overall power consumption is available for water heating. If a combination of hot water and space heating is chosen then a maximum of 72% is available for water heating and at least 22% for space heating.

Summary

1. The basic purpose of material and energy balance is a) to quantify all the material, energy and waste streams in a process or a system and b) to find out the difference between calculated/designed values and measured/actual values thereby making it possible to identify previously unknown losses and emissions.
2. Material and energy balances can be worked out quantitatively knowing the amounts of materials entering into a process, and the nature of the process.
3. Material and energy balances are important, since they make it possible to identify and quantify previously unknown losses and emissions.
4. These balances are useful for monitoring the improvements made in an ongoing project, and while evaluating cost benefits by the top management. Inefficient use of raw materials and energy in

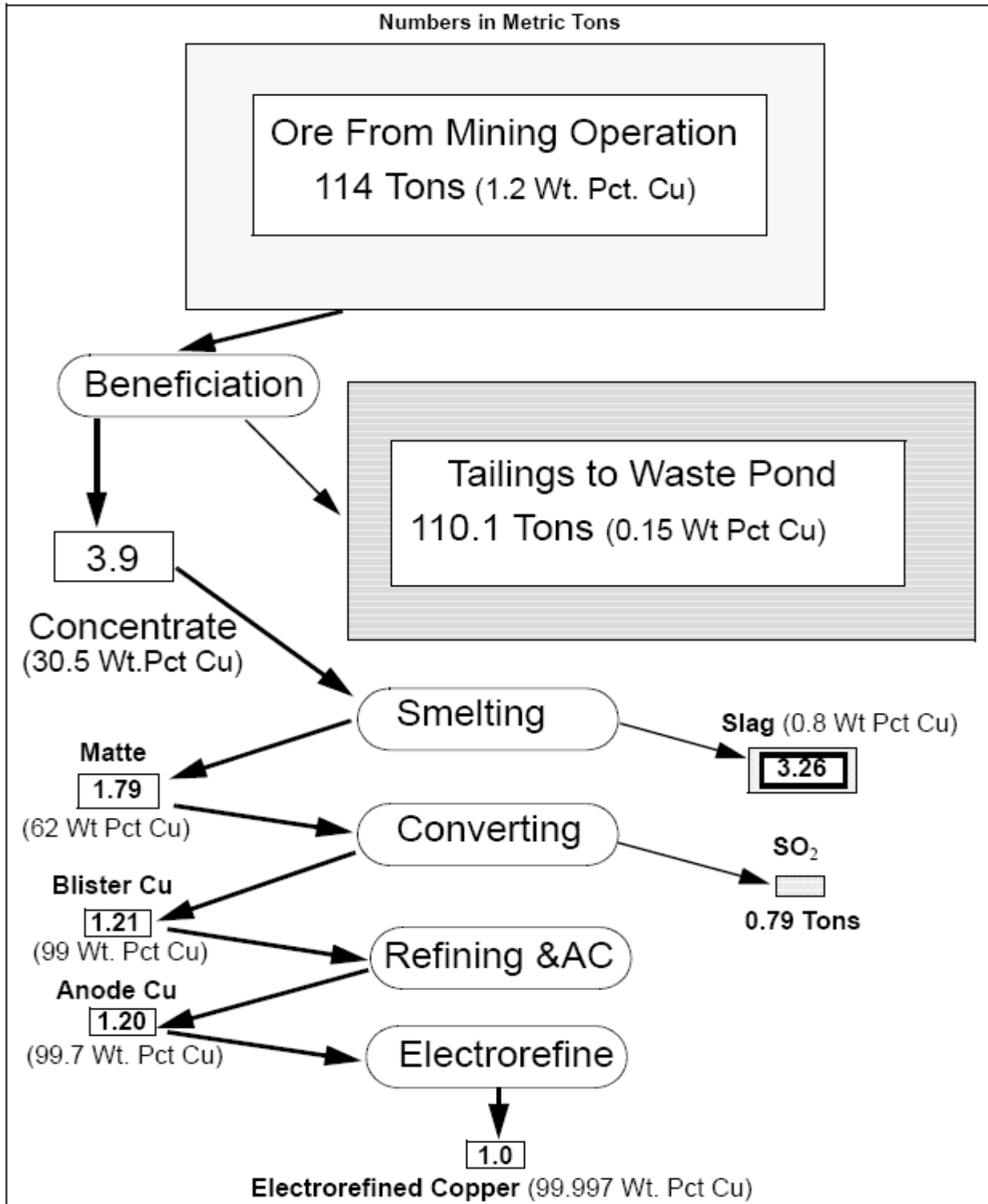
production processes are reflected as wastes. This makes the top management to take quick remedial actions.

5. Material and energy balances take the basic form: Content of inputs = content of products + wastes/losses + changes in stored materials.
6. Energy includes heat energy (enthalpy), potential energy (energy of pressure or position), kinetic energy, work energy, chemical energy. It is the sum over all of these that is conserved.

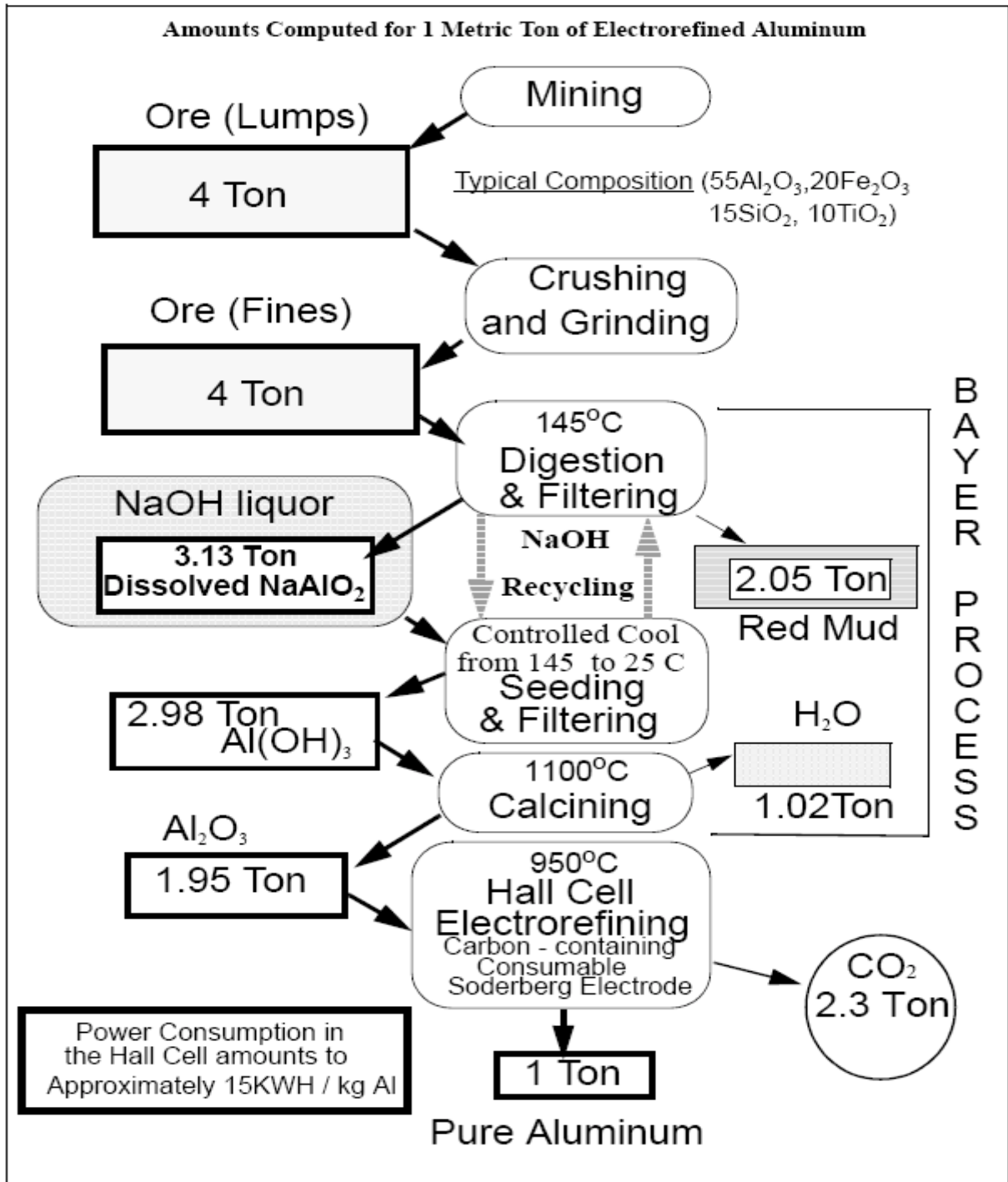
The objective of M&E balance is to assess the input, conversion efficiency, output and losses and when used in conjunction with diagnosis, it is a powerful tool for establishing the basis for improvements and potential savings.

Annexure -1

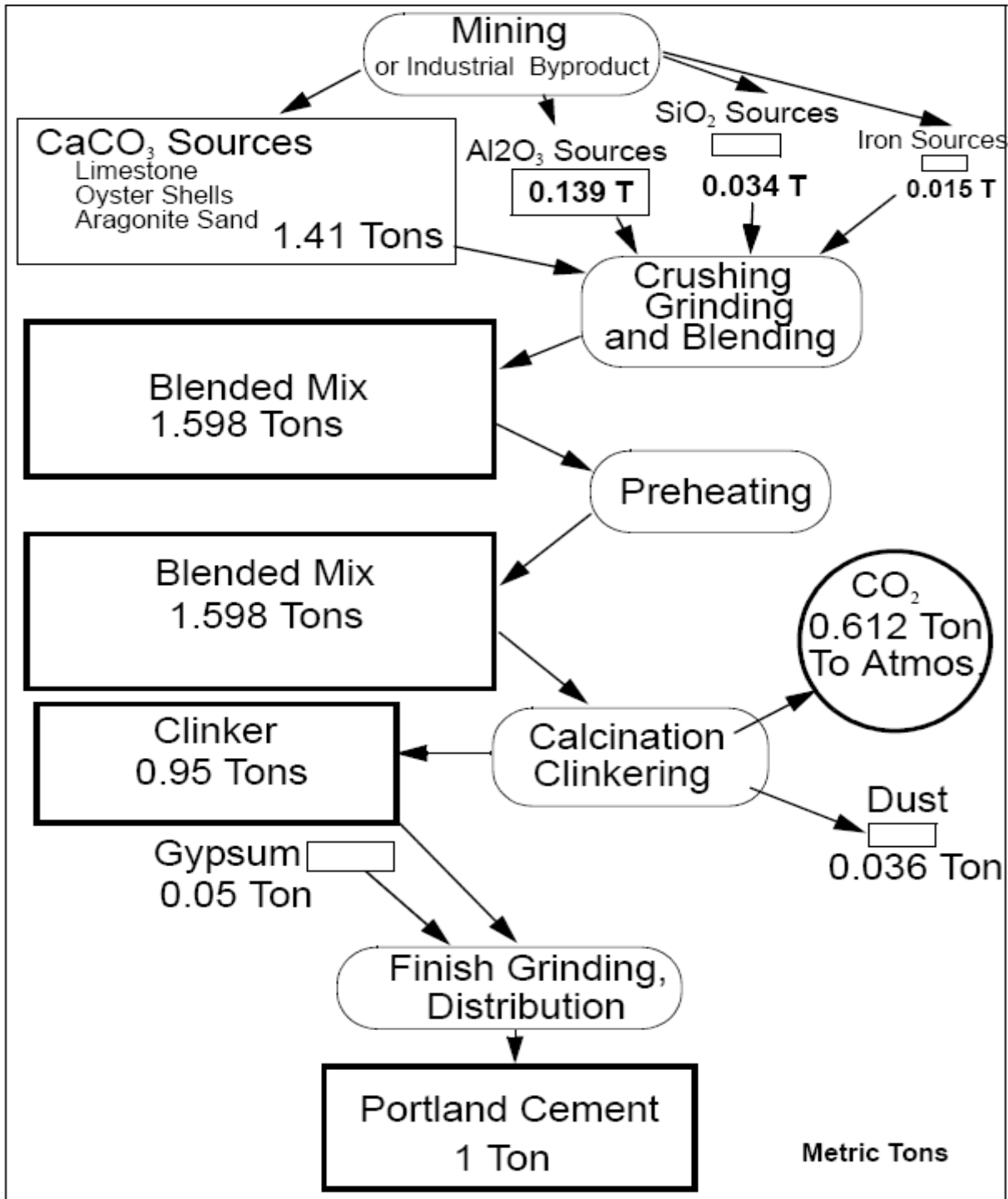
Material Flow Diagram for Copper Production



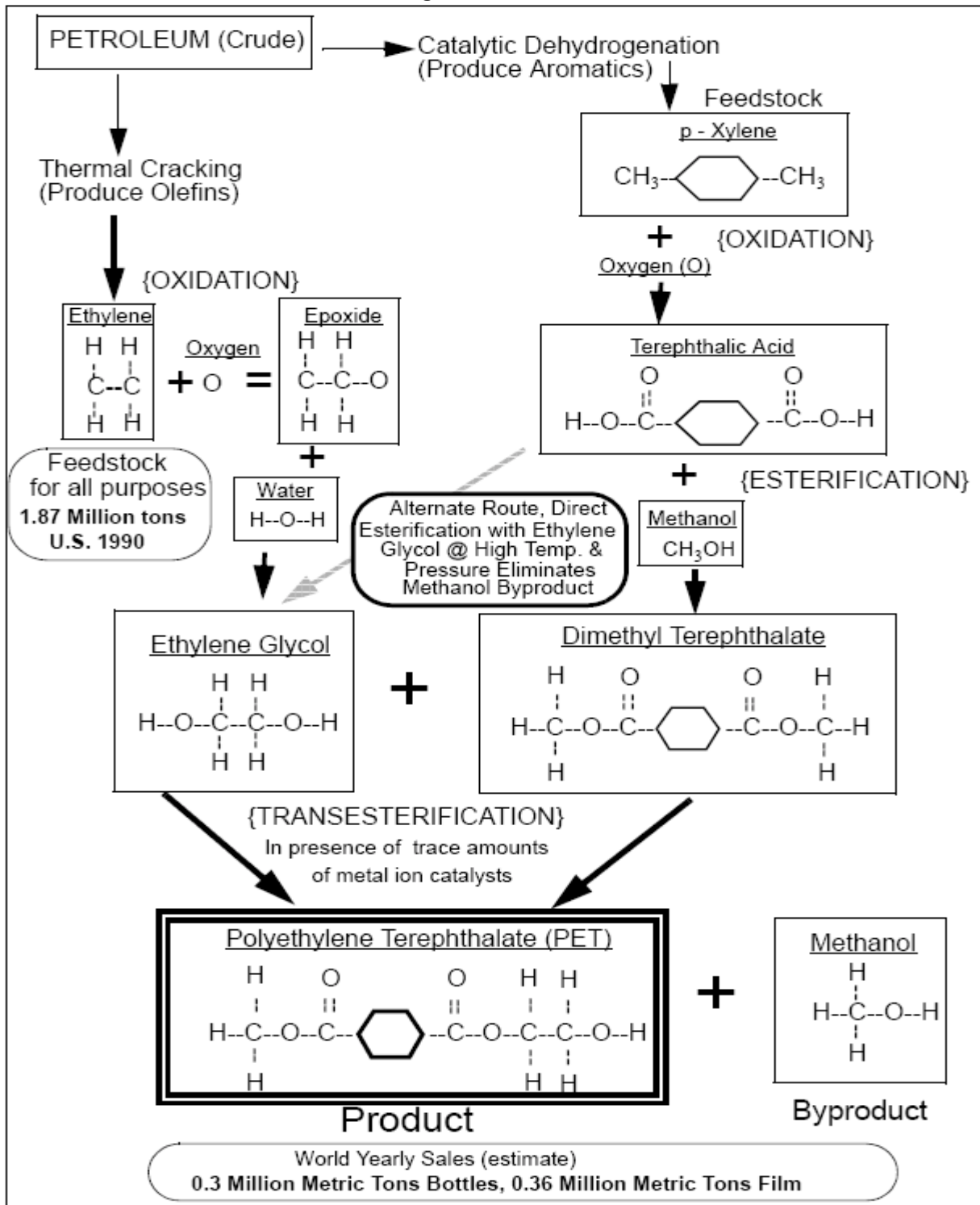
Material Flow Diagram for Aluminum Production



Material Flow Diagram for Portland cement Production



Material Flow Diagram for PET Plastic Production



Material Flow Diagram for Paper Production

