PDHonline Course K101 (3 PDH)

Heating and Cooling of Agitated Liquid Batches: Isothermal Medium

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Course Content

As stated previously, there are many industrial instances whereby it is desirable to estimate the time required to heat or cool a volume of liquid that is being agitated in a tank or reactor vessel. Two common industrial process equipment configurations found are represented in Figures 1 and 2. (All figures located at the end of the Course Outline text.)

Figure 1 represents an arrangement for heating a batch liquid by a medium which is passed through a jacket that surrounds the section of the vessel containing the liquid. Saturated steam is indicated in this example as the heating medium.

Figure 2 represents a batch liquid being cooled using a submerged coil mounted inside the tank within the liquid volume. A Freon based refrigerant, R-22, is indicated in this example as the cooling medium.

In both examples, the volume of batch liquid is well mixed by an internally mounted mixing device. Typically this device is a propeller or turbine blade connected to a shaft that is driven by an electric motor at a fixed speed. The result of the “well mixed” volume of liquid is that this assumes that the liquid physical properties, i.e., temperature, heat capacity, etc., of the liquid are uniform throughout the entire volume, (Assumption 3).

The heating or cooling medium to be used will be under isothermal conditions, defined as $\mathbf{T_{heat}}$ or $\mathbf{T_{cool}}$ respectively. By definition then, the inlet and outlet temperature of the medium is the same, (Assumption 5).

In the case of batch heating, energy is transferred from the heating medium to the cooler liquid batch, see Figure 3. For the temperature of the medium to remain constant, the energy must originate from the phase change of saturated vapor to the saturated liquid. In the case of batch cooling, Figure 4, energy is transferred from the hot batch to the cooling medium. For the temperature of the medium to remain constant, the energy must originate from the phase change of saturated liquid to the saturated vapor, (Assumption 6).

The origin of the application equation starts with a differential heat balance across the medium and liquid. In order to account for heat flow from the higher temperature to the lower temperature the course will evolve an equation each for heating and cooling applications. It will be also be assumed that there is no liquid being vaporized or crystallized during the heating or cooling event, (Assumption 4).

Definition
isothermal = the fluid remains at a constant temperature.

Simplifying assumptions were used in the development of the heating and cooling application equations. These are summarized here:

1. The specific heat of the batch liquid, Cp_{batch} , is constant and will be estimated at the average of the initial and final temperatures.
2. The overall heat transfer coefficient, $U_{jacket\ or\ coil}$, is constant over the entire heat transfer surface area, $A_{jacket\ or\ coil}$.
3. The agitation produces a uniform fluid temperature throughout the batch liquid mass.
4. No phase changes occur in the batch liquid.
5. The heating and cooling media have constant inlet temperatures and flow rates.
6. The heating and cooling media undergo isothermal phase changes.
7. Heat losses to the environment are negligible.

Heating of agitated liquid batches with isothermal medium:

For a mass of liquid M_{batch}, at a specific heat, Cp_{batch}, the heat energy of the fluid, Q, is defined as:

$$Q = M_{batch} * Cp_{batch} * T_{batch} \qquad (1)$$

where:

M_{batch} = mass of the batch liquid being heated or cooled, expressed as lbs.

Cp_{batch} = heat capacity of the liquid batch, expressed as Btu/lb.-F.

T_{batch} = the temperature of the batch liquid at any time, expressed as deg F.

Q = heat transferred, Btu/hr.

The specific heat of the batch liquid will be assumed to be constant. The specific heat used in the application equation will be estimated at the average of the initial and final batch temperatures, (Assumption 1). Per unit time, θ , the change in heat energy of the liquid with respect to changing temperature is given by:

$$dQ/ d\theta = M_{batch} * Cp_{batch}\ dT_{batch}/ d\theta \qquad (2)$$

where:

θ_{hours} = the time calculated to heat or cool the batch liquid from its initial to final temperature, expressed as hours.

At any time, for a given heat transfer area, A, and a transfer coefficient, U, the heat released by the medium and transferred to the batch liquid through the vessel wall is also equal to Q, assuming negligible heat losses to the environment, (Assumption 7). We will also make a simplifying assumption that the overall heat transfer coefficient is constant over the entire transfer area, (Assumption 2). The heat transfer rate is then defined as:

$$Q = U * A * \Delta T \qquad (3)$$

where:

medium = the fluid which supplies the heating and cooling energy to the vessel jacket or tank coil.

A_{jacket} = the jacket heat transfer area utilized by the heating or cooling medium, expressed as sq. ft of available transfer surface, to transfer heat to/from the batch liquid.

A_{coil} = the coil tube heat transfer area utilized by the heating or cooling medium, expressed as sq. ft of available transfer surface, to transfer heat to/from the batch liquid.

U_{jacket} = the overall heat transfer coefficient used in conjunction with the jacket heat transfer area, expressed as Btu/hr-sq. ft.-F.

U_{coil} = the overall heat transfer coefficient used in conjunction with the coil tube heat transfer area, expressed as Btu/hr-sq. ft.-F.

It should be noted that calculations which are for jacketed vessel heating the appropriate terms, U_{jacket} and A_{jacket}, would be substituted for in the above equation. Like wise, for calculations which are intended to represent vessels with submerged heating coils, the terms representing U_{coil} and A_{coil} would be used.

The temperature driving force is defined as :

$$\Delta T = (T_{heat} - T_{batch}) \qquad (4)$$

where:

T_{heat} = the isothermal temperature of the heating medium, expressed as deg F.

Equating (2) and (3) yields:

$$M_{batch} * Cp_{batch}\, dT_{batch} / d\theta = U * A * \Delta T \qquad (5)$$

and substituting equation (4) into (5) yields:

$$M_{batch} * Cp_{batch}\, dT_{batch} / d\theta = U * A * (T_{heat} - T_{batch}) \quad (6)$$

Rearranging similar differential terms and grouping together the constants yields:

$$dT_{batch} / (T_{heat} - T_{batch}) = (U * A / M_{batch} * Cp_{batch})\, d\theta \qquad (7)$$

This can be integrated between the time limits for θ as:

$$\int dT_{batch} / (T_{heat} – T_{batch}) = \int (U * A / M_{batch} * Cp_{batch}) \, d\,\theta \qquad (8)$$

where:

$\theta = 0$, at the start of the cycle which corresponds to $T_{initial}$

θ_{final} = at the finish of the cycle which corresponds to T_{final}.

T_{final} = the final temperature of the batch liquid, expressed as deg F.

$T_{initial}$ = the initial temperature of the batch liquid, expressed as deg F.

and results in:

$$Ln [(T_{heat} - T_{initial})/(T_{heat} - T_{final})] = (U*A* \theta_{hours}) / (M_{batch}*Cp_{batch}) \qquad (9)$$

Rearranging for the time variable yields the following application equation:

$$\theta_{hours} = Ln [(T_{heat} - T_{initial})/(T_{heat} - T_{final})] * (M_{batch}*Cp_{batch})/(U*A) \qquad (10)$$

Cooling of agitated liquid batches with an isothermal medium:

For a mass of liquid M_{batch}, at a specific heat, Cp_{batch}, the heat energy of the fluid is defined as:

$$Q = M_{batch} * Cp_{batch} * T_{batch} \qquad (11)$$

Per unit time, the change in heat energy of the liquid with respect to changing temperature is given by:

$$dQ/\,d\theta = M_{batch} * Cp_{batch}\, dT_{batch}/\, d\theta \qquad (12)$$

at any time, for a given heat transfer area, A, and a transfer coefficient, U, the heat transferred to the batch liquid through the vessel wall is defined as:

$$Q = U * A * \Delta T \qquad (13)$$

It should be noted that calculations which are for jacketed vessel heating the appropriate terms, U_{jacket} and A_{jacket} , would be substituted for in the above equation. Like wise, for calculations which are intended to represent vessels with submerged heating coils, the terms representing U_{coil} and A_{coil} would be used.

The temperature driving force is defined as :

$$\Delta T = (T_{batch} – T_{cool}) \qquad (14)$$

where:

T_{cool} = the isothermal temperature of the cooling medium, expressed as deg F.

Equating (12) and (13) yields:

$$M_{batch} * Cp_{batch}\, dT_{batch}\, /d\theta = U * A * \Delta T \qquad (15)$$

Substituting equation (14) into (15) yields:

$$M_{batch} * Cp_{batch}\, dT_{batch}\, /d\theta = U * A * (T_{batch} – T_{cool}) \quad (16)$$

Rearranging similar differential terms and grouping together the constants gives:

$$dT_{batch} / (T_{batch} – T_{cool}) = (U * A/M_{batch} * Cp_{batch})\, d\theta \qquad (17)$$

This can be integrated between the time limits for θ as:

$$\int dT_{batch} / (T_{batch} – T_{cool}) = \int (U * A / M_{batch} * Cp_{batch})\, d\,\theta \qquad (18)$$

where:

$\theta = 0$, at the start of the cycle which corresponds to $T_{initial}$

θ_{final} = at the finish of the cycle which corresponds to T_{final}.

and results in:

$$Ln\ [\ (T_{initial} – T_{cool})/(T_{final} – T_{cool})\ ]\ =\ (U * A * \theta_{hours}) / (M_{batch} * Cp_{batch}) \qquad (19)$$

Rearranging for the time variable yields the following application equation:

$$\theta_{hours} = Ln\ [\ (T_{initial} – T_{cool})/(T_{final} – T_{cool})\ ]\ * (M_{batch} * Cp_{batch})/(U * A) \qquad (20)$$

Application Equation Summary

For isothermal heating of agitated liquid batches:

$$\theta_{hours} = Ln\ [\ (T_{heat} - T_{initial})/(T_{heat} - T_{final})\ ] * (M_{batch} * Cp_{batch})/(U * A) \qquad (10)$$

For isothermal cooling of agitated liquid batches:

$$\theta_{hours} = Ln\ [\ (T_{initial} - T_{cool})/(T_{final} - T_{cool})\ ] * (M_{batch} * Cp_{batch})/(U * A) \qquad (20)$$

Physical Property Data Table 1.

COMPOUND	PHYSICAL PROPERTY	REFERENCE #
Water	Liquid Density @ 110 deg F= 8.27 lbs./gal	5
Water	Liquid Heat Capacity @ 110 = 1.0 Btu/lb.-F	3
Steam	Saturation Temperature @ 164.7 psia = 366 deg F	5
R-22	Saturation Temperature @ 34.7 psia = -5 deg F	6

Example #1

It is proposed to heat 5,000 gallons of cold water in an agitated jacketed vessel vented to the atmosphere. Plant steam at 150 psig is to be used as the heating medium. It is given that the heat transfer area of the jacket is 200 sq. ft. and that an overall heat transfer coefficient of 100 could be assumed. Calculate the time required to heat the water volume from 70 to 150 deg F.

Step #1.1: Calculate the average temperature of the water.

$Tbatch_{avg}$ = the average temperature of the batch liquid, expressed as deg F. This will be used as the temperature for determining the physical properties.

$$T_{avg} = (T_{initial} + T_{final})/2$$

$$T_{avg} = (70 + 150)/2 = 110 \text{ deg F}$$

$$T_{avg} = 110 \text{ deg F}$$

Step #1.2: Determine the density of the water at the average temperature.

From Table 1, the density at 110 deg F is 8.27 lbs./gal

Step #1.3: Calculate the mass of water to be heated.

Mass = Volume * Density

M_{batch} = 5,000 * 8.27 = 41,350 lbs.

Step #1.4: Determine the heat capacity of the water being heated.

From Table 1, the heat capacity, Cp_{batch} is 1.0 Btu/lb.-F

Step #1.5: Determine the steam heating medium temperature at the plant supply pressure with pressure in absolute units.

Saturated steam: a typical heat medium in the chemical process industries. Its chemical formula is H_2O. The normal boiling point for water is 212 deg F at atmospheric pressure, 14.7 psia or 0 psig. When water is at this temperature and pressure both vapor and liquid are saturated and in equilibrium with each other. Typically Steam Tables are used to determine saturation for water temperatures at various pressures.

Pressure = 150 + 14.7 = 164.7 psia.

From Table 1, the steam temperature, T_{heat} , is 366 deg F

Step #1.6: Calculate the time required to heat the mass of water given the data obtained in steps #1.1 through #1.5 above using application equation (10).

$$\theta_{hours} = Ln\ [\ (T_{heat} - T_{initial})/(T_{heat} - T_{final})\ ] * (M_{batch}*Cp_{batch})/(U*A)$$

$$\theta_{hours} = Ln\ [\ (366 - 70)/(366 - 150)\ ] * (41{,}350*1.0)/(100*200)$$

$$\theta_{hours} = Ln\ [\ 1.37\ ] * 2.07$$

θ_{hours} = 0.65 hours, or 39 minutes.

Example #2

It is proposed to cool 5,000 gallons of water in an agitated vessel vented to the atmosphere that is equipped with an internal coil. Cooling inside the coil will be done using refrigerant R-22 at 20 psig as the medium. It is given that the heat transfer area of the coil is 450 sq. ft. and that an overall heat transfer coefficient of 50 could be assumed. Calculate the time required to cool the water volume from 170 to 50 deg F.

Step #2.1: Calculate the average temperature of the water.

$$T_{avg} = (\ T_{initial} + T_{final}\ )/2$$

T_{avg} = (170 + 50)/2 = 110 deg F

T_{avg} = 110 deg F

Step #2.2: Determine the density of the water at the average temperature.

From Table 1, the density at 110 deg F is 8.27 lbs./gal

Step #2.3: Calculate the mass of water to be heated.

Mass = Volume * Density

M_{batch} = 5,000 * 8.27 = 41,350 lbs.

Step #2.4: Determine the heat capacity of the water being heated.

From Table 1, the heat capacity, Cp_{batch} , is 1.0 Btu/lb.-F

Step #2.5: Determine the coolant temperature at the plant supply pressure with pressure in absolute units.

Freon 22: a typical refrigerant that is commonly used under isothermal conditions. It commercial name is refrigerant 22 or R-22. Its chemical formula is $CHClF_2$. The normal boiling point for this compound is –41.4 deg F. See reference 6 for the saturation temperature at various pressures.

Pressure = 20 + 14.7 = 34.7 psia

From Table 1, the coolant temperature, T_{cool} , is -5 deg F

Step #2.6: Calculate the time required to cool the mass of water given the data obtained in steps #2.1 through #2.5 above using application equation (20).

$$\theta_{hours} = \text{Ln}\left[\ (T_{initial} - T_{cool})/(T_{final} - T_{cool})\ \right] * (M_{batch} * Cp_{batch})/(U*A)$$

$$\theta_{hours} = \text{Ln}\left[\ (170 – (-5))/(50 – (-5))\ \right] * (41{,}350 * 1.0)/(50 * 450)$$

$$\theta_{hours} = \text{Ln}\left[\ 3.18\ \right] * 1.84$$

$$\theta_{hours} = 2.1 \text{ hours, or } 128 \text{ minutes}$$

Figure 1. Agitated Liquid in a Vessel with a heating Jacket

Vessel Data:
Volume= 5,500 gals
Jacket Heat Transfer Area = 200 sq. ft.
Overall Transfer Coefficient = 100 Btu/hr-sq ft-F

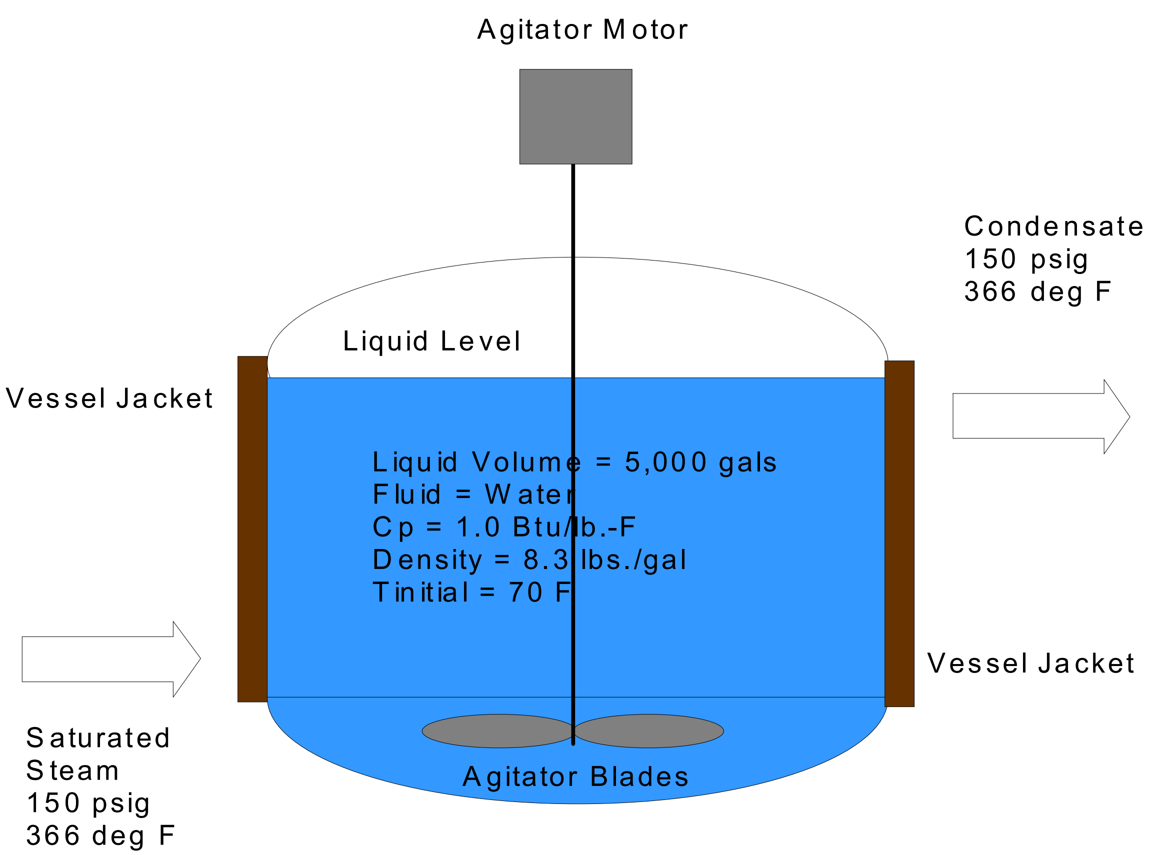

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Figure 2. Agitated liquid in a Vessel with a Submerged Cooling Coil

Vessel Data:
Volume= 5,500 gals
Coil Heat Transfer Area = 450 sq. ft.
Overall Transfer Coefficient = 50 Btu/hr-sq ft-F

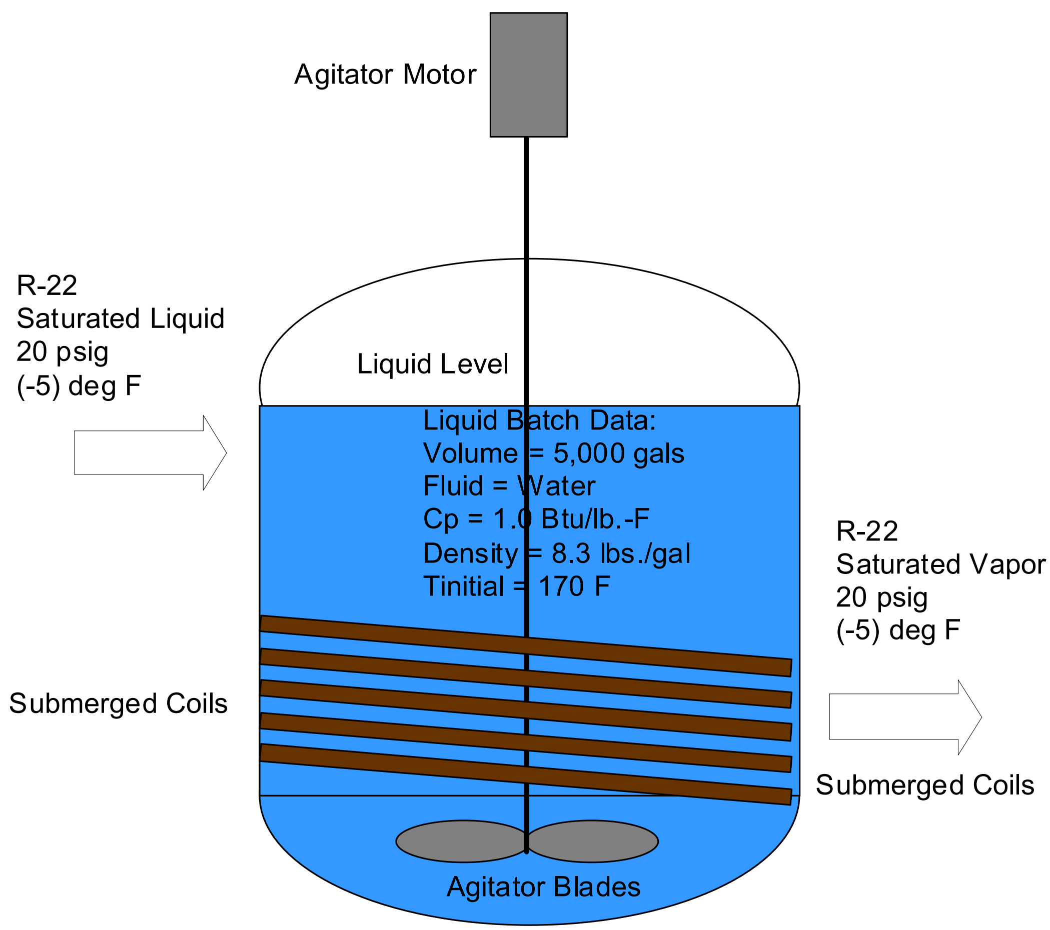

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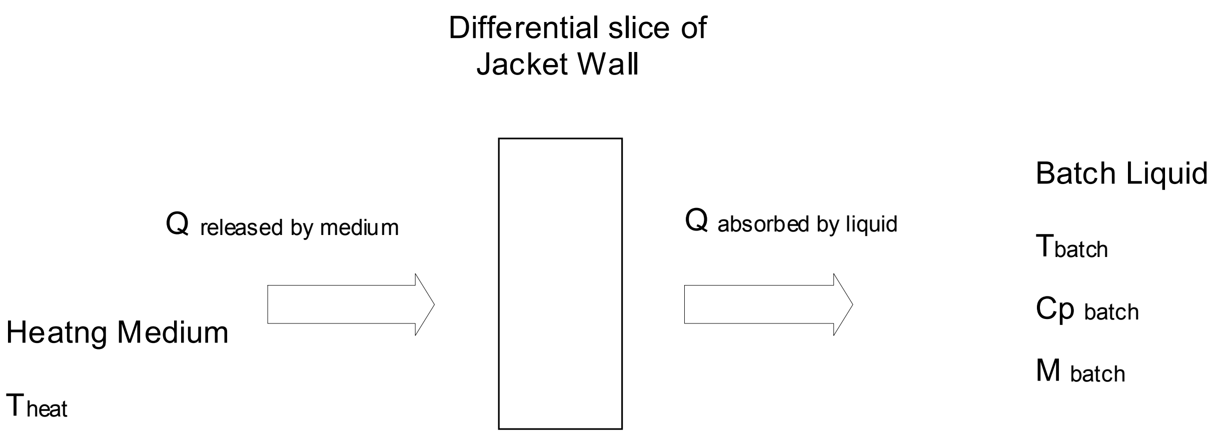
Figure 3. Differential representation of heat flow for batch liquid heating

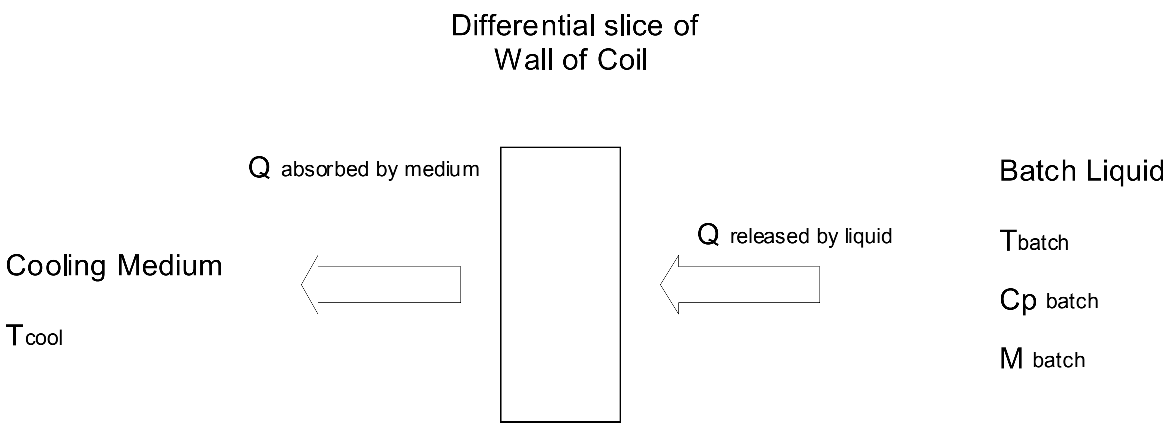


Area = A_{jacket}
Heat transfer coefficient = U_{jacket}

$T_{heat} > T_{batch}$

Temperature driving force = $T_{heat} - T_{batch}$

Figure 4. Differential representation of heat flow for batch liquid cooling



Area = A_{coil}
Heat transfer coefficient = U_{coil}

$T_{cool} < T_{batch}$

Temperature driving force = $T_{batch} - T_{cool}$

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