



PDHonline Course K103 (2 PDH)

Mechanical Agitator Power Requirements for Liquid Batches

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2012

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Mechanical Agitator Power Requirements for Liquid Batches

Course Content

Mechanical Agitator Power Requirements for Liquid Batches can be determined from the energy balance equation and empirical correlations. Figure 1 represents a typical industrial system used for mixing a batch liquid (Figures are located at the end of the Course Content example). The mixing of batch liquids, as studied in this course has two important industrial applications. The first is to blend miscible liquids, such as methyl alcohol and water. The second is to promote heat transfer between the batch liquid and a coil or jacket.

The volume of batch liquid is mixed by an internally mounted mixing device called an agitator impeller. Typically the impeller is a single propeller or turbine blade connected to a shaft that is driven by an electric motor at a fixed speed.

There are two classes of impeller agitators: axial-flow and radial-flow, and the mixing characteristics are shown in Figure 2. Axial-flow impellers generate currents parallel with the axis of the impeller shaft. Radial-flow impellers generate currents in a tangential or radial direction to the axis of the impeller shaft.

Within the two classes of impellers there exist three main types of impeller design. Figure 3 shows the types and variation of common impellers. The three types are: propeller, turbine, and paddle. The three main types are utilized in about 95 percent of most batch liquid agitation systems.

Standard propellers have three blades, but can be two-bladed, four-bladed, or encased by a circular guard. A revolving propeller traces out a helix in the fluid. One full revolution moves the liquid a fixed distance. The ratio of this distance to the propeller diameter is known as the pitch. Propellers are a member of the axial class of impeller agitators.

Turbines are six or more blades mounted at the end of the agitator shaft. They are a member of the radial class of impeller agitators. Turbine diameter is typically 30 to 50 percent of the vessel diameter.

Paddles are two or four blades mounted on the end of the agitator shaft. They are a subset of the radial class of impeller agitators. Typically the impeller diameter of paddles is 50 to 80 percent of the tank diameter.

The agitator impeller is, in essence, a pumping device operating without the typical confines of a casing or directed inlet and outlet flows. As the impeller blade rotates, fluid is forced outward from the blade tip. The movement force is a vector that can

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be described by radial and tangential velocity components. If we assume that the tangential liquid velocity is some fraction k , of the blade-tip velocity then:

$$V_{u2} = k * u_2 = k * \pi * D * N \quad (1)$$

where:

V_{u2} = tangential component velocity of the liquid leaving the blade tip.

u_2 = velocity of the impeller blade tip.

D = diameter of the impeller, feet.

N = rotational speed of the impeller.

The volumetric flow rate through the radial sweep of the impeller, see Figure 6, is expressed as:

$$q = V_{r2} * A_p \quad (2)$$

where:

V_{r2} = radial component velocity of the liquid leaving the blade tip.

A_p = area of the cylinder swept out by the impeller blade tips, sq. ft.

By definition:

$$A_p = \pi * D * W \quad (3)$$

where:

q = volumetric flow rate through the impeller, cu. ft/sec.

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W = width of the impeller blades, ft.

By defining the angle at which the fluid leaves the impeller as β_2 , it can be shown that:

$$V_{r2} = (u_2 - V_{u2}) * \tan \beta_2 \quad (4)$$

substituting equations (1), (2), and (3) into equation (4) and rearranging yields:

$$q = \pi^2 * D^2 * N * W * (1 - k) * \tan \beta_2 \quad (5)$$

For geometrically similar impellers W is proportional to D . For given k and β_2 values the flow can be expressed as:

$$q \propto N * D^3 \quad (6)$$

The ratio of equations (5) and (6) is defined as the Flow Number, N_Q , and written as:

$$N_Q = q / (N * D^3) \quad (7)$$

For marine propellers β_2 values can be considered constant. For turbines β_2 values are a function of the relative impeller and tank diameter size. Typically the design of agitated baffled vessels uses the Flow Number to be:

For marine propellers: $N_Q = 0.5$

For six bladed- turbines: $N_Q = 0.93 * T / D$

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where :

T = diameter of the vessel or tank.

The power input to a frictionless impeller is derived in detail from a Bernoulli energy balance equation, in reference 1 page 190. This relationship is

$$P_{\text{Delivered}} * g_c = m' * \omega * r_2 * V_{u2} \quad (8)$$

where:

$P_{\text{Delivered}}$ = Power delivered in ft-lb_f/sec.

g_c = Newton's-law proportionality factor, 32.2 ft-lb_m/lb_f-sec².

m' = mass flow rate in lb./sec.

ω = angular velocity in radians/sec.

r_2 = radius of impeller

The right hand side of equation (8) can be re-written as equation (9) by substituting for $\omega = 2 * \pi * N$, and $r_2 = D/2$:

$$P_{\text{Delivered}} * g_c = m' * \pi * D * N * V_{u2} \quad (9)$$

Substituting equation (1) and $m' = \rho_B * q$ into equation (9) yields:

$$P_{\text{Delivered}} * g_c = \pi^2 * \rho_B * q * k * D^2 * N^2 \quad (10)$$

where:

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ρ_B = batch liquid density at the average batch temperature, lb./cu. ft.

Substituting equation (5) into equation (10) and rearranging into dimensionless form yields:

$$(P_{\text{Delivered}} * g_c) / (N^3 * D^5 * \rho_B) = \pi^4 * (W / D) * (1 - k) * \tan \beta_2 \quad (10)$$

The left-hand side of equation (10) is called the power number, N_P :

$$N_P = (P_{\text{Delivered}} * g_c) / (N^3 * D^5 * \rho_B) \quad (11)$$

$$P_{\text{Delivered}} = N_P * (N^3 * D^5 * \rho_B) / g_c \quad (12)$$

The right-hand side of equation (10) indicates that the power number, N_P , depends on the ratio of width of the impeller blades to impeller diameter, W / D , as well as k and β_2 . The values of k and β_2 vary with many parameters of the system and cannot be predicted theoretically. They must be found by experiment.

As a side note, the power number, flow number and k are related by dividing equation (10) by the term $(\rho_B * D^5 * N^3)$ and using the definitions of equations (7) and (11) to result in:

$$k = \pi^2 * (N_P / N_Q) \quad (13)$$

Calculation of Agitated Liquid Batch Power Numbers

To determine the power delivered to a given fluid at a set rotational speed, several other variables are also required. These variables relate to the vessel geometry and other “shape” parameters. A number of benchmark studies were done by J. H. Rushton, Reference # 10, in the 1950’s to correlate power number with know vessel

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geometry and shape parameters. The basis for the power correlations was done using a modified Reynolds number that was specific for agitated liquid batches. The agitated liquid batch Reynolds number indicates the flow of momentum associated with the bulk motion of the fluid. The Reynolds number is defined for agitated batch liquids as:

$$N_{Re} = (D^2 * N * \rho_B / \mu_B) \quad (14)$$

μ_B = viscosity of the batch liquid at the average batch liquid temperature, lb./hr-ft.

D = agitator cross sectional diameter measured tip to tip, ft.

N = agitator speed, revolutions per hour.

Figure 4 is a typical power correlation curve for agitated liquid batches. Both of the axes are defined in logarithmic scale. The x-axis is the Reynolds number as calculated by equation (14). The y-axis is the experimentally determined power number. The plot indicates that at various stages of flow regimes; laminar, transitional, and turbulent, the power number varies with the Reynolds number. It can be seen from the plot, which as the Reynolds number is increased from early stages of turbulence to increasingly larger Reynolds numbers, the power number becomes constant. This course will focus on the mechanical power requirements for liquid batches in the turbulent flow regime, as defined as:

$$N_{Re} > 10,000 \quad (15)$$

All power numbers reported within this course will deal with correlation results in the turbulent flow regime and be reported as a constant. Power number constants are listed in Table 2 for the various impeller types shown in Figure 3.

Figure 5 and Table 1 are references for the various shape parameters that impact the power number. The example that follows will demonstrate how to determine motor power requirements for agitated liquid batches.

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The selection of an electric motor required to deliver the design power to an application consists of two additional steps after the power delivered has been calculated from equations (12) and (14).

$$N_{Re} = (D^2 * N * \rho_B / \mu_B) \quad (14)$$

$$P_{Delivered} = N_P * (N^3 * D^5 * \rho_B) / g_c \quad (12)$$

Given known values for fluid properties, impeller diameter, and rotational speed, along with the power number from the appropriate correlation, the first step is an adjustment for friction losses. These losses are from the loss of usable energy in the form of work through the motor, shaft gearing, and bearings.

$$P_{Motor} = P_{Delivered} / (100 - \%Losses) \quad (16)$$

The second step round up motor horsepower to closest higher standard size. Table 4, Three-Phase Electric Motor Sizes, is provided for the student's use.

$$P_{Design} = P_{Motor} \quad (17)$$

A rule of thumb that has been utilized as a rough guide for quickly estimating mechanical agitator power requirements for water is given in Table 5.

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Application Equations

Mechanical agitator power requirements for liquid batches are calculated by determining the power number, N_P , for a given system and correcting for motor, gearing and bearing losses. Design specification of the motor is then determined by selecting the closest higher standard size.

The system turbulence is checked by calculating the Reynolds number for the agitation system. Adequate turbulence is defined at Reynolds numbers greater than 10,000. Shape factors S1 through S9 are calculated using the mixing vessel geometry as defined in Table 1 and shown in Figure 5. A power number is then obtained by comparing the shape factor results with published power correlations. Table 2 is provided for six types of typical agitator impellers.

$$N_{Re} = (D^2 * N * \rho_B / \mu_B) \quad (14)$$

$$P_{Delivered} = N_P * (N^3 * D^5 * \rho_B) / g_c \quad (12)$$

$$P_{Motor} = P_{Delivered} / (100 - \%Losses) \quad (16)$$

$$P_{Design} = P_{Motor} \quad (17)$$

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Table 1. Shape Factor Descriptions and Definitions.
For use with Table 2 and Figure 5.

Shape Factor	Ratio	Description
S1	T/D	Tank Diameter/Impeller Diameter
S2	Z/D	Height of Impeller above vessel floor/Impeller Diameter
S3	L/D	Length of impeller blade/Impeller Diameter
S4	D/W	Impeller Diameter/Width of blade
S5	T/B	Tank Diameter/Baffle width
S6	H/D	Liquid depth in vessel/Impeller Diameter
S7	#	Number of impeller blades
S8	degrees	pitch/angle
S9	#	Number of baffles

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Table 2. Power Numbers, N_P , for Various Impeller Types and Shape Parameters

Impeller Type	Ref #	S1	S2	S3	S4	S5	S6	S7	S8	S9	Power Number, PO = constant. $N_{Re} > 10,000$
Propeller	6	3	0.75-1.3	ND	ND	10	2.7-3.9	3	pitch=D	4	0.35
Propeller	6	3	0.75-1.3	ND	ND	10	2.7-3.9	3	pitch=2D	4	0.9
Pitched blade turbine	4	ND	ND	ND	8	12	ND	6	45 deg	4	1.5
Straight full blade turbine	4	ND	ND	ND	8	12	ND	6	0 deg	4	3
Curved full blade turbine	8	ND	ND	ND	8	ND	ND	ND	0 deg	ND	2.7
Flat blade turbine	6	3	0.75-1.3	0.25	5	6	2.7-3.9	6	0 deg	4	7
Flat blade turbine	6	3	0.75-1.3	0.25	5	10	2.7-3.9	6	0 deg	4	6
Flat blade turbine	6	3	0.75-1.3	0.25	5	25	2.7-3.9	6	0 deg	4	4
Flat paddle	6	3	0.75-1.3	1	4	10	2.7-3.9	2	0 deg	4	1.8
ND= No Data.											

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Table 3. Physical Property Data

COMPOUND	PHYSICAL PROPERTY	REF
Water	Temperature = 80 deg F	
	Liquid Density = 8.56 lbs./gal	5
	Liquid Viscosity = 0.877 centipoise	5

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Table 4. Three-Phase Electric Motor Horsepower Sizes

MOTOR HP
1
1.5
2
3
5
7.5
10
15
20
25
30
40
50
60
75
100
125
150
200
250
300
REF #11

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Table 5. Rule of Thumb Power Ratios for Water

AGITATION	POWER RATIO HP/1000 GALS
Mild	0.5-1.0
Vigorous	2-3
Intensive	4-10
	Reference #1

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Example #1

It is proposed to mix a batch of water at 80 deg F in an agitated vessel similar to the one shown in Figure 1. Calculate the power required, in horse power, for an agitation speed of 60 rpm. Use an approximate loss of efficiency of 20% for the motor, gearbox, and bearings. Can the mixing be considered intensive under these conditions? Equipment details are given below:

Equipment Data:

Vessel Diameter (T) = 9.0 ft.

Impeller Diameter (D) = 3.0 ft.

Impeller type = Flat blade turbine.

of blades = 6

Blade width (W) = 7 inches.

Blade length (L) = 9 inches.

Number of Baffles = 4; equally spaced.

Baffle width (B) = 11 inches.

Liquid Depth (H) = 9.0 feet.

Impeller height above vessel bottom (Z) = 3 feet.

Step #1.1: Determine the agitated batch liquid physical properties:

- a. From Table 3. the physical properties of the batch liquid at 80 deg F are:

ρ_B = Liquid Density = 8.56 lbs./gal

μ_B = Liquid Viscosity = 0.877 centipoise

Converting liquid density to lbs./cu. ft. requires multiplying the density from Table 3 by 7.48 and results in a liquid density of $\rho_B = 64.0$ lbs./cu. ft.

Converting viscosity to lb./hr-ft units requires multiplying the viscosity from Table 3 by 2.42 and results in a liquid viscosity of $\mu_B = 2.12$ lb./hr-ft.

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Step #1.2: Determine the agitator speed in appropriate units for Reynolds number calculation:

N = revolutions per hour (rph). The agitation speed is given in the example as 60 revolutions per minute, which when multiplied by 60 minutes/hr yields:

$$N = 60 \text{ rpm} * 60 = 3,600 \text{ rph}$$

Step #1.3: Determine the agitated batch liquid Reynolds number using equation (14):

$$N_{Re} = \text{Reynolds number for agitated liquids} = (D^2 * N * \rho_B / \mu_B).$$

$$N_{Re} = (3.0^2 * 3,600 * 64.0 / 2.12)$$

$$N_{Re} = 977,035. \text{ This exceeds requirement for equation (15)}$$

$$N_{Re} > 10,000 \quad (15)$$

Step #1.4 Determine the values for the shape factors S_1 through S_9 .

Using Table 1 and Figure 5 along with the information given in the example yields:

$$S_1 = T/D = 9/3 = 3$$

$$S_2 = Z/D = 3/3 = 1$$

$$S_3 = L/D = (9/12)/3 = 0.25$$

$$S_4 = D/W = 3/(7/12) = 5$$

$$S_5 = T/B = 9/(11/12) = 9.8$$

$$S_6 = H/D = 9/3 = 3$$

$$S_7 = 6$$

$$S_8 = 0 \text{ Degrees, no pitch}$$

$$S_9 = 4$$

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Step #1.5: Determine the appropriate power number N_P from the three flat blade turbine impellers listed in Table 2.

The flat blade impeller that most closely resembles the shape factors is the second one listed. Its power number N_P is:

$$N_P = 6$$

Step #1.6: Determine the agitator speed in appropriate units for power calculation:

N = revolutions per sec (rps). The agitation speed is given in the example as 60 revolutions per minute, which when divided by 60 seconds/minute yields:

$$N = 60 \text{ rpm} / 60 = 1 \text{ rps}$$

Step #1.7: Determine Power Number from the appropriate correlation, and calculate the uncorrected Power required. The correlations calculate the delivered Power to the fluid. This must be corrected back to the agitator motor and account for efficiency losses in the motor, gearbox, and bearings.

$$P_{\text{Delivered}} = (N_P \rho_B * N^3 * D_A^5) / G_C$$

$$P_{\text{Delivered}} = 6 * 64 * 1^3 * 3^5 / 32.2$$

$$P_{\text{Delivered}} = 2,898 \text{ ft-lb./sec}$$

or using 550 ft-lb./sec per Hp,

$$P_{\text{Delivered}} = 5.3 \text{ Hp}$$

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Step #1.8: Determine the motor Power required using estimated losses at 30%:

$$P_{\text{Motor}} = P_{\text{Delivered}} / (1 - \text{Losses})$$

$$P_{\text{Motor}} = 5.3 / (1 - 0.20)$$

$$P_{\text{Motor}} = 6.6 \text{ Hp}$$

Step #1.9: Round up motor horsepower to closest higher standard size from Table 4:

$$P_{\text{Motor}} = 7.5 \text{ Hp.}$$

Step #1.10: Determine power to volume ratio:

$$P_{\text{Motor}} = 7.5 \text{ Hp}$$

$$\text{Volume} = 5,500 \text{ gallons or } 5.5 \text{ thousand gallons}$$

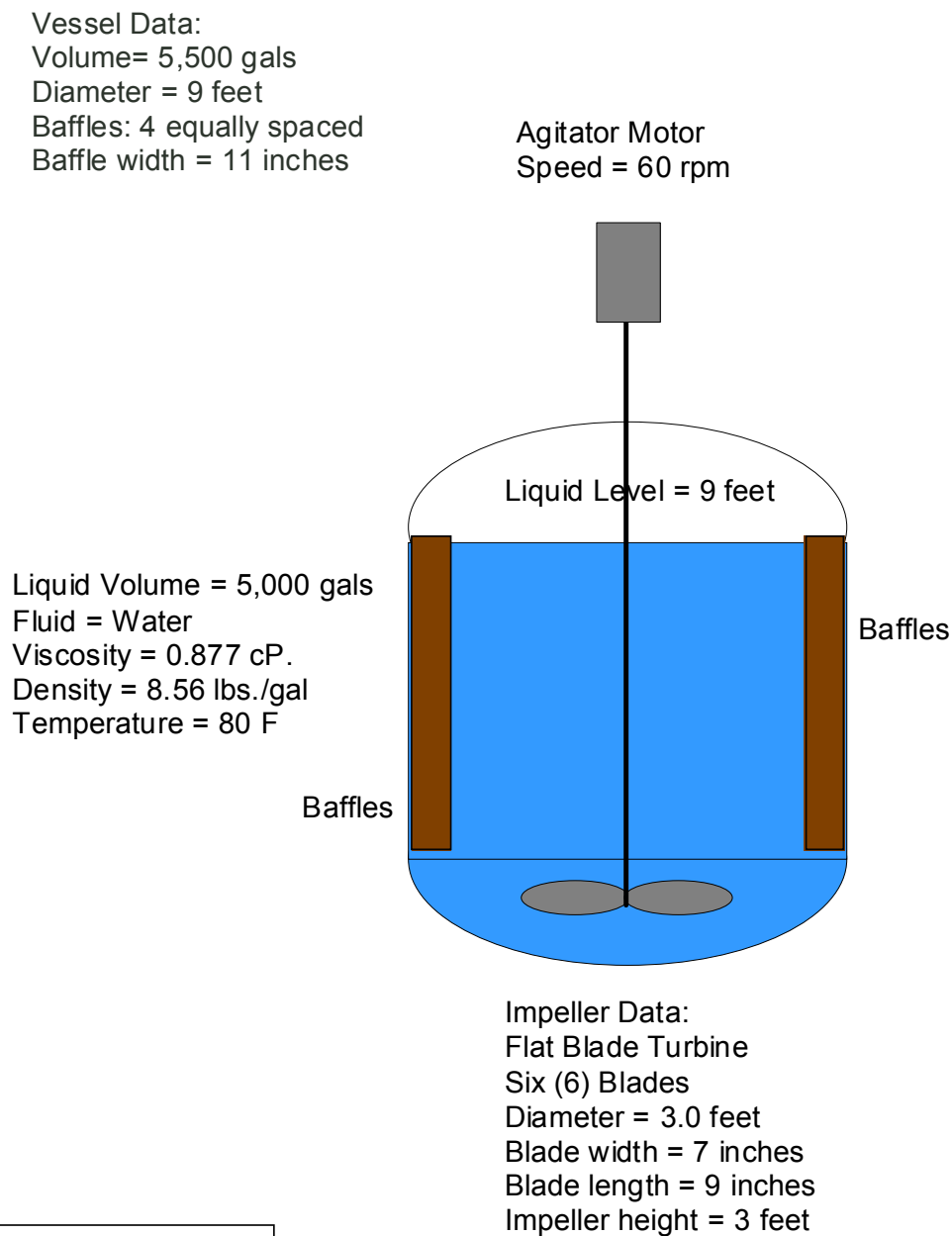
$$\text{Power Ratio} = 7.5 \text{ Hp} / 5.5$$

$$\text{Power Ratio} = 1.4$$

Can the mixing be considered intensive under these conditions? No. From Table 5 the agitation is between mild and vigorous.

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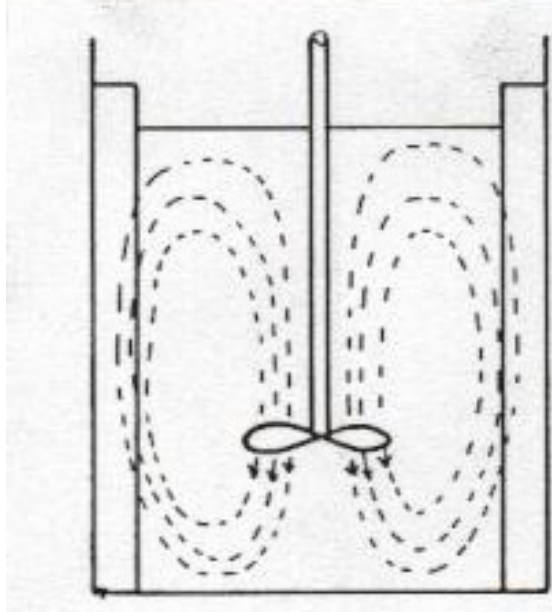
Figure 1. Agitated Liquid Batch



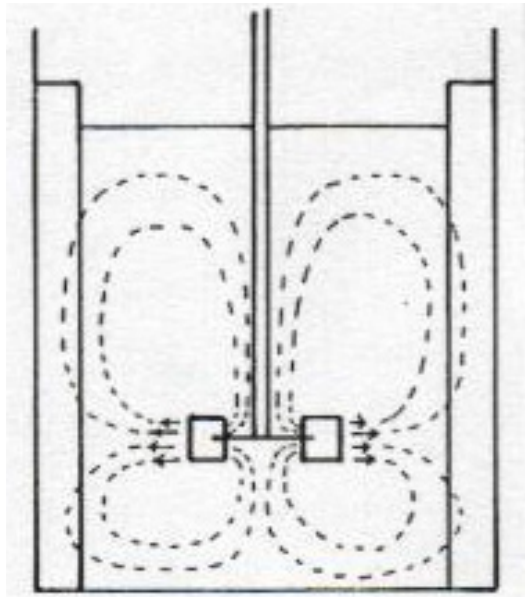
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Figure 2. Agitated Liquid Batch Flow Patterns



Axial Flow Pattern for
Marine Impeller



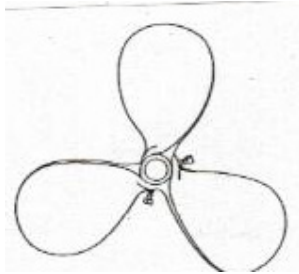
Radial Flow Pattern for
Flat Blade Turbine Impeller

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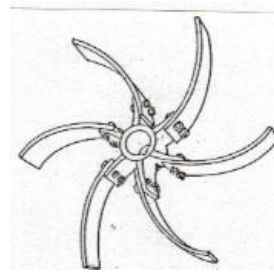
Reference #9.

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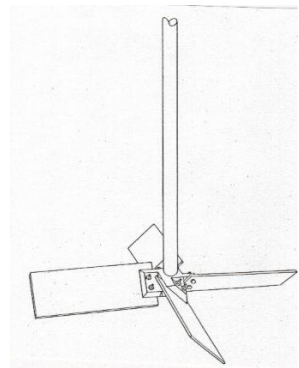
Figure 3. Impeller Types



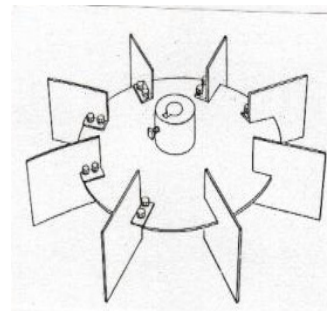
A. Marine Propeller, Ref #4.



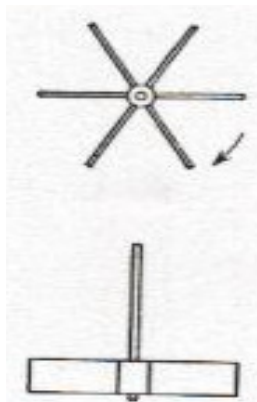
B. Curved Full Blade Turbine, Ref #4.



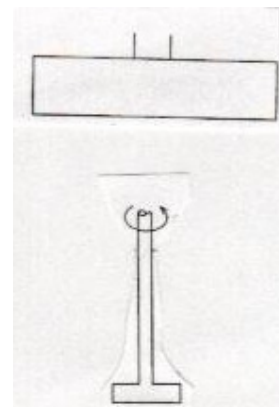
C. Pitched Blade Turbine, Ref #4.



D. Flat Blade Turbine, Ref #4.



E. Straight Full Blade Turbine, Ref #1.

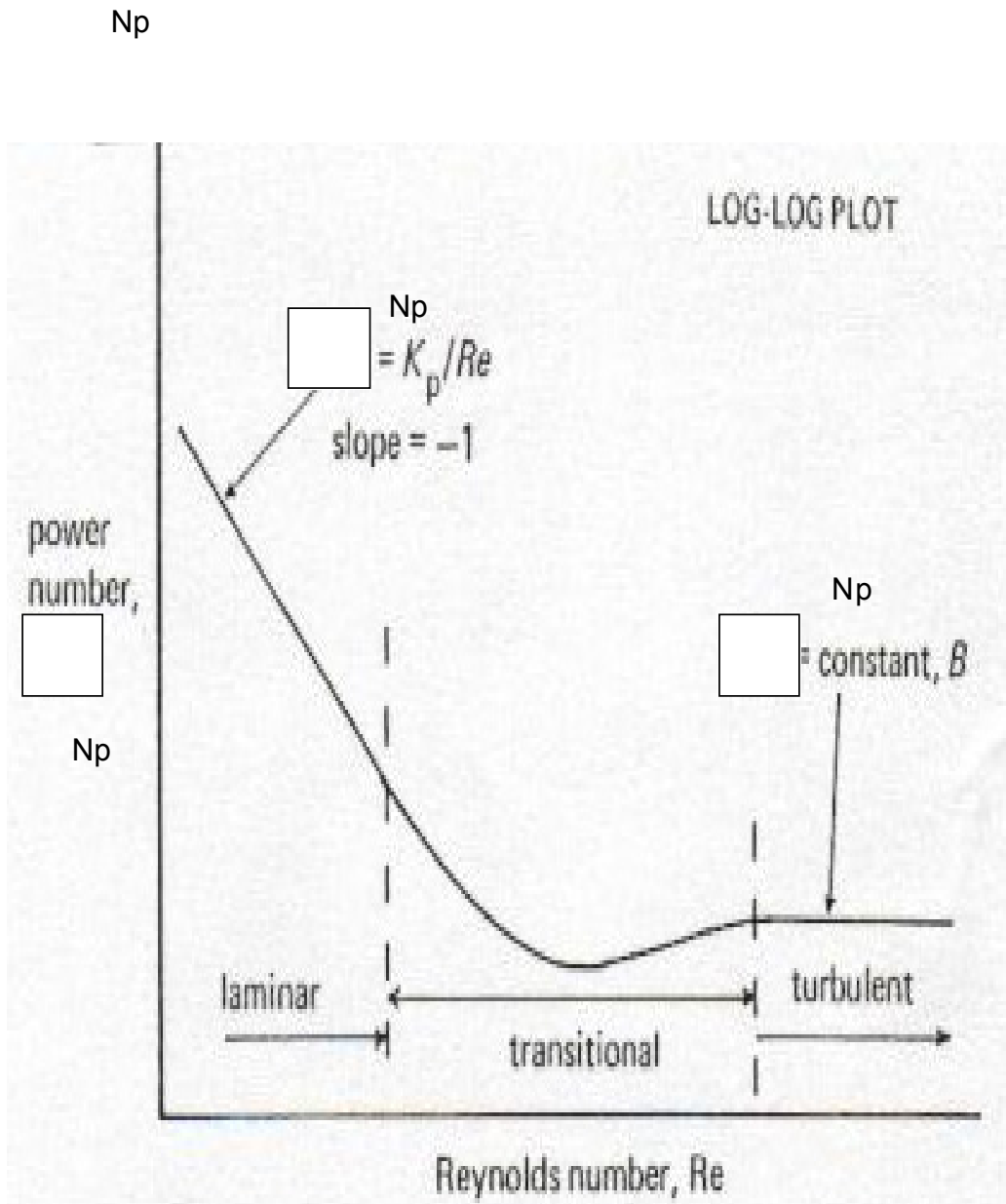


F. Flat Paddle, Ref #6.

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Figure 4. Typical Power Curve for Agitator Impellers

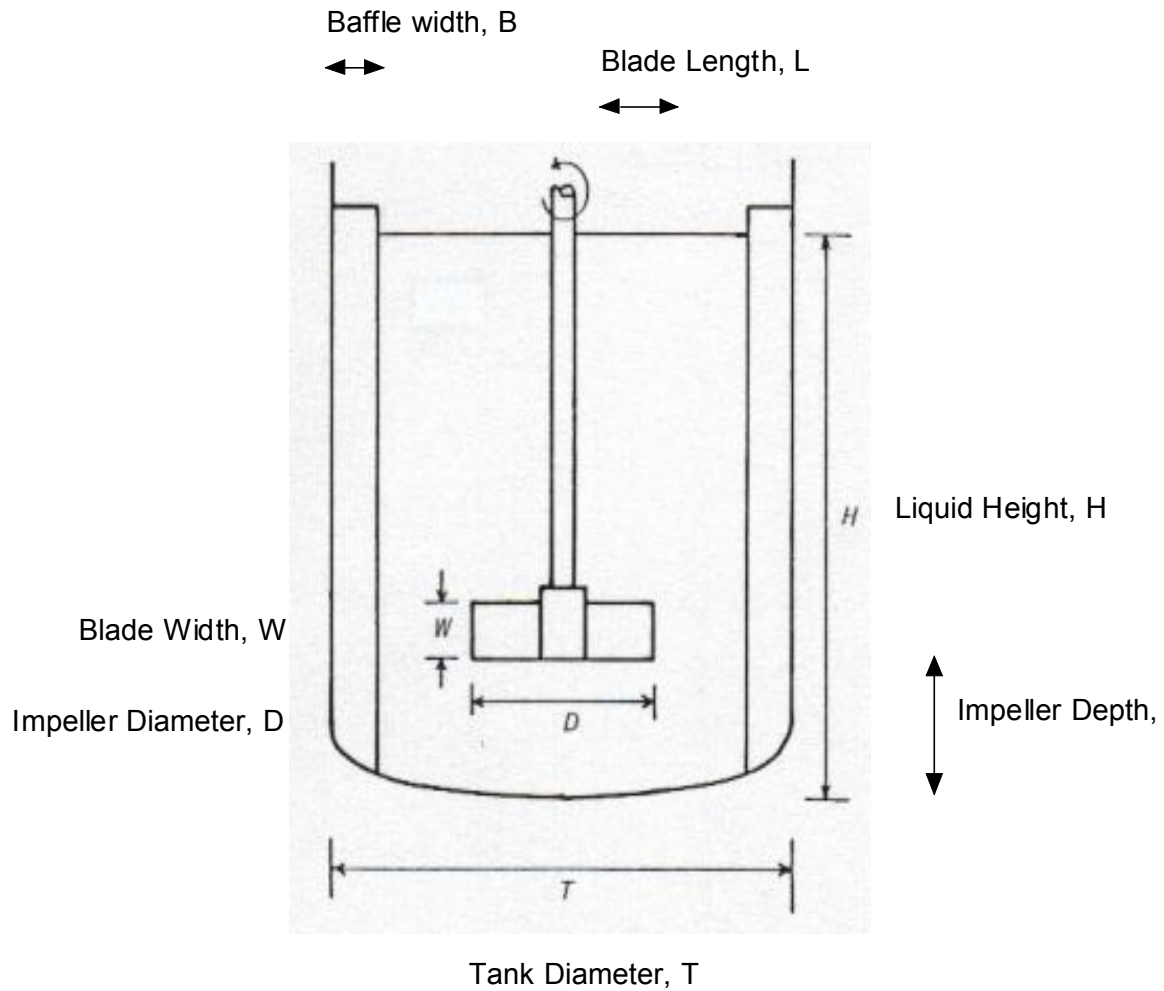


Reference #9.

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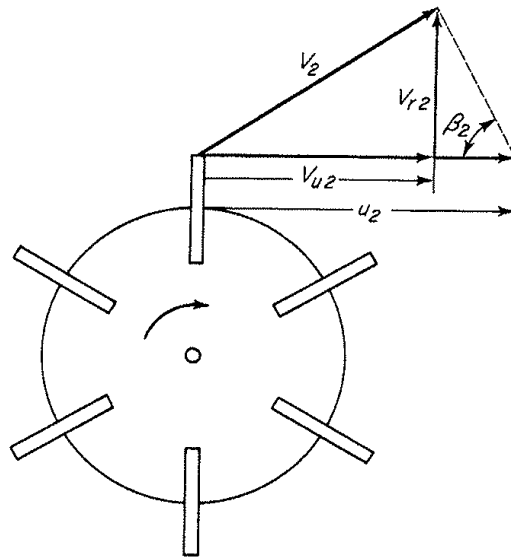
Figure 5. Agitated Liquid Batch Shape Parameters



Reference #9.

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Figure 6. Velocity vectors at tip of Turbine Impeller Blade



It may be shown that for a given impeller, the equation below relates k and β_2

$$\tan \beta_2 = (2k/(1 - k))^{0.5}$$

Reference #1.