PDHonline Course M229 (4 PDH)

Basic Fundamentals of Gear Drives

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2012

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Course Content

GEARS

Gears are mechanical elements which are used for transmitting a synchronous motion directly between shafts; these may be parallel or at any skew axis. Gears are the most durable and rugged of all mechanical drives and can be used for transmitting high power with high efficiency (up to 99% in the case of parallel shafts).

The design of meshing gears is complex especially if an optimum solution is required in an application. British, American and ISO standards describe analytical techniques in much detail but they tend to be rather daunting to the inexperienced and infrequent gear designer. Manufacturers' catalogues, in contrast, often do not give adequate design information to enable a thorough verifying analysis to be performed on the gears they produce. Frequently a simplified analytical selection procedure is given but for a more comprehensive explanation it is necessary to make reference to the appropriate standard for details.

This course provides a basic overview of the gear drives and transmission.

Gear Classification

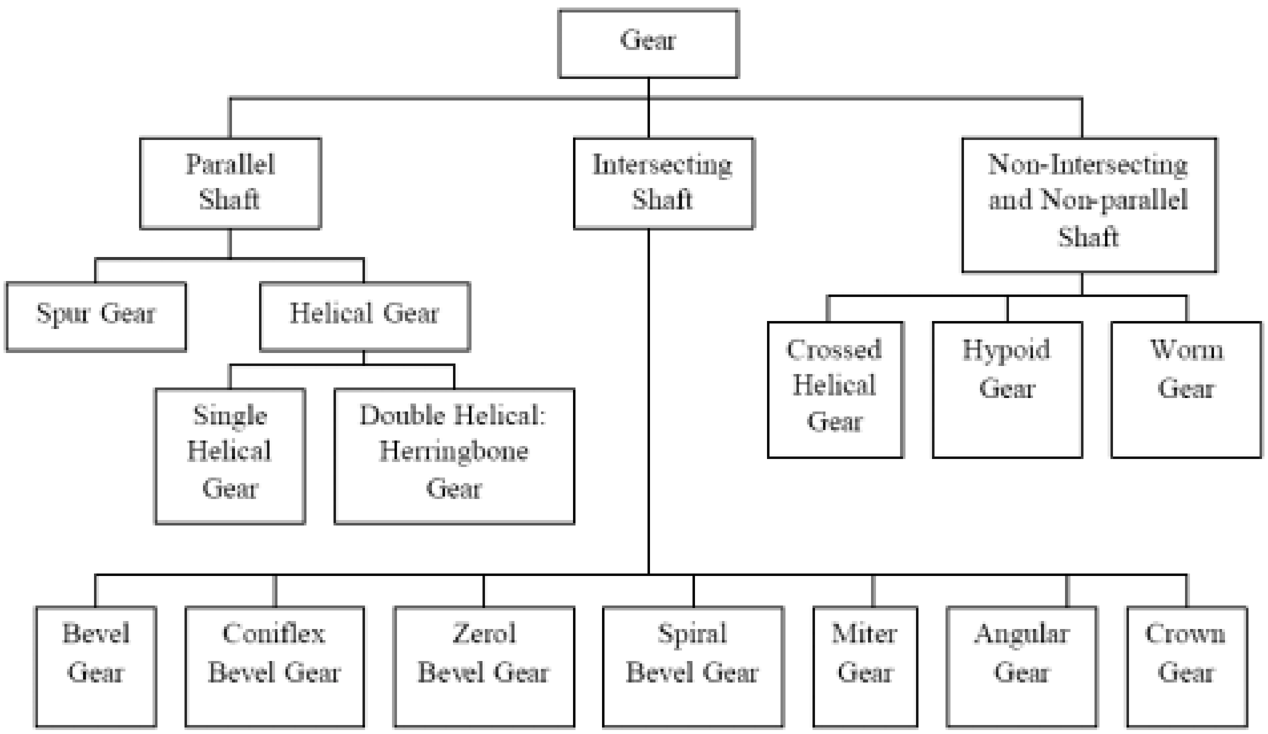
Gears may be classified according to the relative position of the axes of revolution. The axes may be a) Parallel, b) Intersecting or c) Non-parallel and non-intersecting.

1) Parallel Gear Shafts - The gears that have parallel shafts are spur gears, helical gears, and herringbone gears. These are the gears that mesh in the same plane. They are used for transmission of rotary motion between parallel shafts. They offer maximum transmission of power and high efficiency. They are used in higher horsepower applications where long-term operational efficiency is more important. They are more expensive to manufacture and create axial thrust. They have higher load capacity and make less noise. They are generally used in clocks, movie projector and car steering. They are widely used for manual transmission.

2) Intersecting Shaft Gears - The gears that have intersecting shaft are bevel gears, straight bevel gears, zerol bevel gears and spiral gears. These gears are designed for the efficient transmission of power and motion between intersecting shafts at an angle. These are ideal for applications requiring high load capacity and are cheaper than the parallel shaft gears. They are used where speed and strength are desirable along with the change in angle of power flow. These gears are widely used in hand drill, locomotives, automobiles, marine applications and all rotorcraft drive system.

3) Neither Parallel nor Intersecting- The gears that have neither parallel nor intersecting shafts are hypoid gears, crossed helical gears and worm gears. These gears provide an effective answer for power transmission applications requiring high-ratio speed reduction in a limited space using non-intersecting shafts. These are ideal for applications requiring only limited load capacity. Lowering ratios easily increases their efficiency. These gears are the cheapest of all gears and are widely used in automobile differentials, electric mixer, speedometer, sprinkler, machine tools, and packaging machinery.

The chart below highlights the gear categories:



Velocity Classification

Gears are further classified according to velocities.

1) Gears are considered to be low velocity type, if their peripheral velocity lies in the range of 1 to 3 m/sec

2) Gears are considered to be medium velocity type, if their peripheral velocity lies in the range of 3 to 15 m/sec

3) Gears are considered to be high velocity type, if their peripheral velocity exceeds 15 m/sec

TYPE OF GEARS

Various types of gears commonly used are described below:

SPUR GEARS

Spur gears connect parallel shafts and are used when shaft rotates in the same plane. These have involute teeth that are parallel to the shaft and can have internal or external teeth. Spur gears are by far the most commonly used gear type commonly available, and are generally the least expensive.

Characteristics:

- The speed and change of the force depends on the gear ratio, the ratio of number of teeth on the gears that are to be meshed.
- They have higher contact ratio that makes them smooth and quiet in operation.
- They are available for corrosion resistant operation and can be used to create large gear reductions.
- They cause no external thrust between gears. They give lower but satisfactory performance.

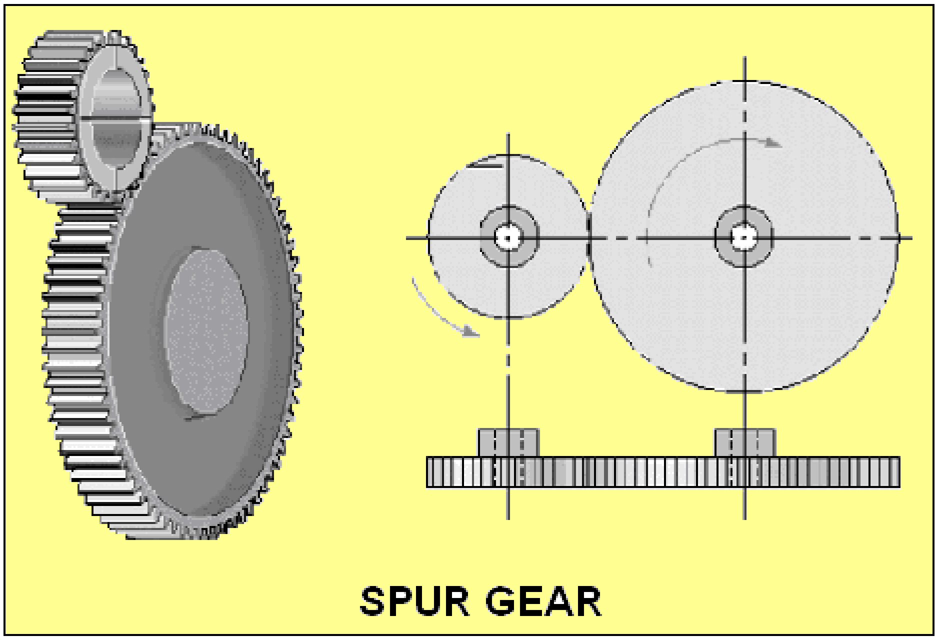
Applications:

- Spur gears are easy to find, inexpensive, and efficient.
- Generally used in simple machines like washing machines, clothes dryer or power winches. They are also used in construction equipment, machine tools, indexing equipment, multi spindle drives, roller feeds, and conveyors.

Limitations:

- Spur gears generally cannot be used when a direction change between the two shafts is required.
- They are not used in certain applications such as automobiles because they produce sound when the teeth of both the gears collide with each other. It also increases stress on the gear teeth.

The basic descriptive geometry for a spur gear is shown in the figure below.



HELICAL GEARS

Helical gears are similar to the spur gear except that the teeth are at an angle to the shaft, rather than parallel to it as in a spur gear. The resulting teeth are longer than the teeth on a spur gear of equivalent pitch diameter. The longer teeth cause helical gears to have the following differences from spur gears of the same size.

Characteristics

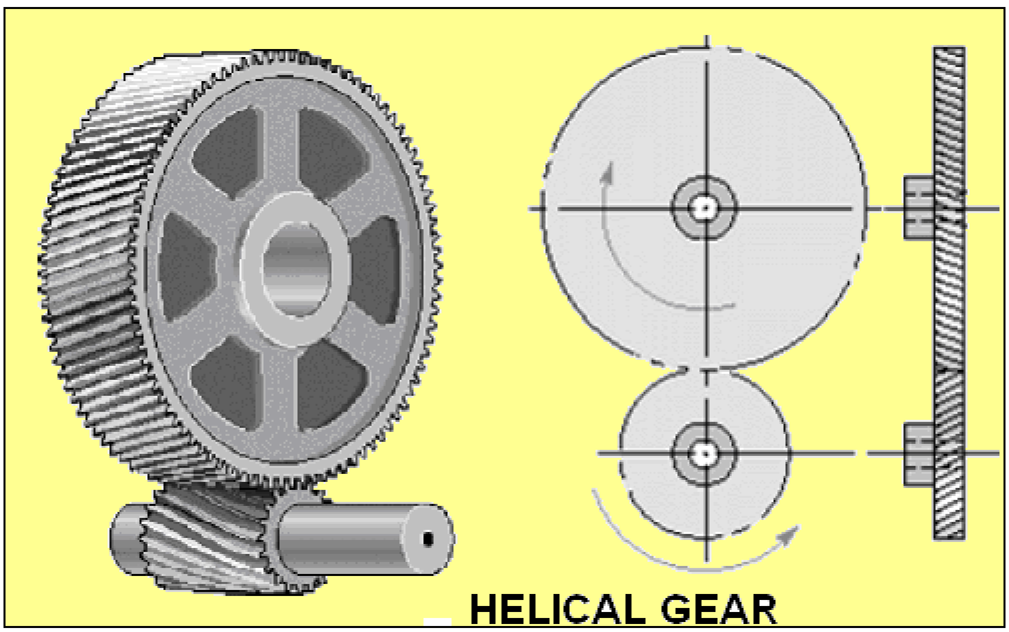
- The basic descriptive geometry for a helical gear is essentially the same as that of the spur gear, except that the helix angle must be added as a parameter.
- The helical teeth result in a greater tooth width which enables increased load and power transmissibility for a given gear size. Also they result in a smoother engagement and so are less noisy in operation than straight spur gears.
- Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily used in parallel shaft applications.
- Tooth strength is greater because the teeth are longer. Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear but the longer surface of contact also reduces the efficiency of a helical gear relative to a spur gear.
- A special application in which helical gears are used is a crossed gear mesh, in which the two shafts are perpendicular to each other.

Applications:

- Helical gears can be used on non parallel and even perpendicular shafts, and can carry higher loads than can spur gears.
- These are highly used in power transmission because they are quieter even at higher speed and are durable. The other possible applications of helical gears are in textile industry, blowers, feeders, rubber and plastic industry, sand mullers, screen, sugar industry, rolling mills, food industry, elevators, conveyors, cutters, clay working machinery, compressors, cane knives and in oil industry.

Disadvantage:

- A disadvantage of helical gear is the resultant thrust along the axis of the gear, which needs to be accommodated by appropriate thrust bearings. This can be overcome by the use of double helical gears having teeth with a 'v' shape.
- These are expensive compared to spur gears and much more difficult to find.



BEVEL GEARS

Bevel gears are primarily used to transfer power between intersecting shafts and are useful when the direction of a shaft's rotation needs to be changed.

Characteristics:

- The teeth of bevel gears are formed on a cone surface.
- These gears permit minor adjustment during assembly and allow for some displacement due to deflection under operating loads without concentrating the load on the end of the tooth.

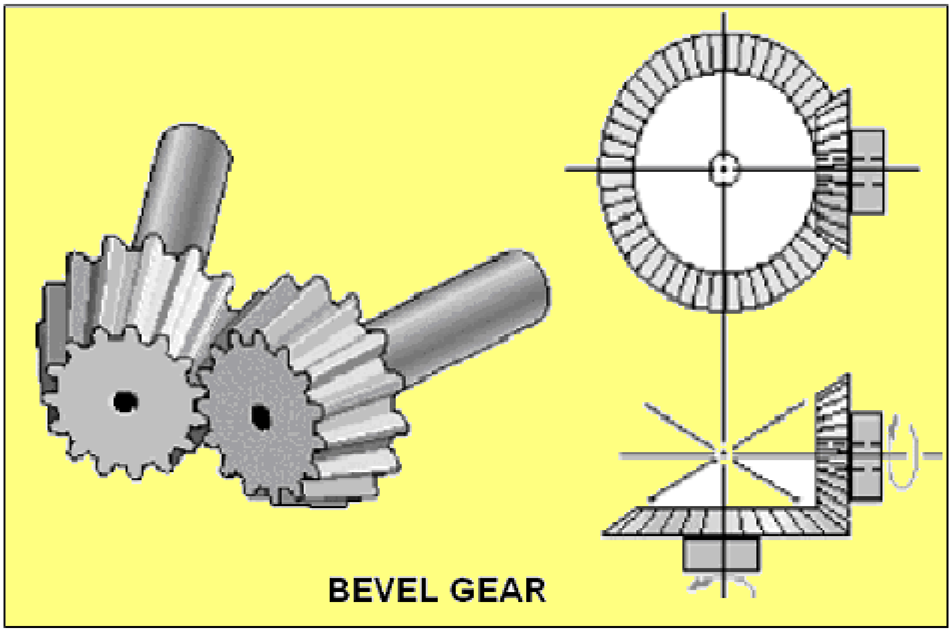
Types:

Bevel gears come in several types. The teeth on bevel gears can be straight, spiral or bevel.

- Standard bevel gears have teeth which are cut straight and are all parallel to the line pointing the apex of the cone on which the teeth are based. In straight bevel gears teeth have no helix angles. In straight when each tooth engages it impacts the corresponding tooth and simply curving the gear teeth can solve the problem. Straight tool bevel gears are generally considered the best choice for systems with speeds lower than 1000 feet per minute: they commonly become noisy above this point.
- Spiral bevel gears are also available which have teeth that form arcs, which gives performance improvements. The contact between the teeth starts at one end of the gear and then spreads across the whole tooth.
- Hypocycloid bevel gears are a special type of spiral gear that will allow nonintersecting, non-parallel shafts to mesh. The hypoid bevel gears can engage with the axes in different planes. This is used in many car differentials. The ring gear of the differential and the input pinion gear are both hypoid. This allows input pinion to be mounted lower than the axis of the ring gear. Hypoid gears are stronger, operate more quietly and can be used for higher reduction ratios. They also have sliding action along the teeth, potentially reducing efficiency.

Applications:

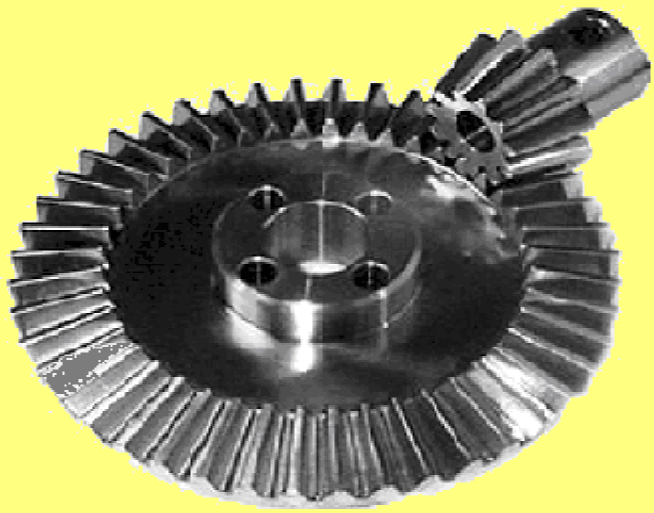
- Excellent choice for intersecting shaft systems. A good example of bevel gears is seen as the main mechanism for a hand drill. As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation. The bevel gears in a hand drill have the added advantage of increasing the speed of rotation of the chuck and this makes it possible to drill a range of materials.
- The bevel gears find its application in locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants and also in on all current rotorcraft drive system.
- Spiral bevel gears are important components on all current rotorcraft drive systems. These components are required to operate at high speeds, high loads, and for an extremely large number of load cycles. In this application, spiral bevel gears are used to redirect the shaft from the horizontal gas turbine engine to the vertical rotor.



MITTER GEARS

Miter gears are bevel gears put together with equal numbers of teeth and axes that are usually at right angles. Miter is the surface forming the beveled end or edge of a piece where a miter joint is made.

- They are designed for the efficient transmission of power and motion between intersecting shafts at right angles.
- They are known for efficient power transfer and durability.
- They give smoother, quieter operation.
- They handle higher speeds and greater torque loads.
- They provide a steady ratio.
- They are used as important parts of conveyors, elevators and kilns.



MITTER GEAR

WORM GEARS

Worm gears are special gears that resemble screws, and can be used to drive spur gears or helical gears. Worm gears, like helical gears, allow two non-intersecting 'skew' shafts to mesh. Normally, the two shafts are at right angles to each other but not in the common plane. A worm gear is equivalent to a V-type screw thread. Another way of looking at a worm gear is that it is a helical gear with a very high helix angle.

Characteristics:

- One very important feature of worm gear meshes is their *irreversibility i.e.* when a worm gear is turned, the meshing spur gear will turn, but turning the spur gear will not turn the worm gear. The resulting mesh is 'self-locking', and is useful in ratcheting mechanisms. The gear cannot turn the worm because the angle on the worm is shallow and when the gear tries to spin the worm, the friction between the two holds the worm in place.
- Worm gear is always used as the input gear. For the operation of worm gear, torque is applied to the input end of the worm shaft by a driven sprocket or electric motor. The worm and the worm shaft are supported by anti-friction roller bearings.
- Worm gears are normally used when a high gear ratio is desired.

Advantages:

- Will tolerate large loads and high speed ratios. These are used when high speed reduction more than 10:1 is required.
- Meshes are self locking (which can be either an advantage or a disadvantage).

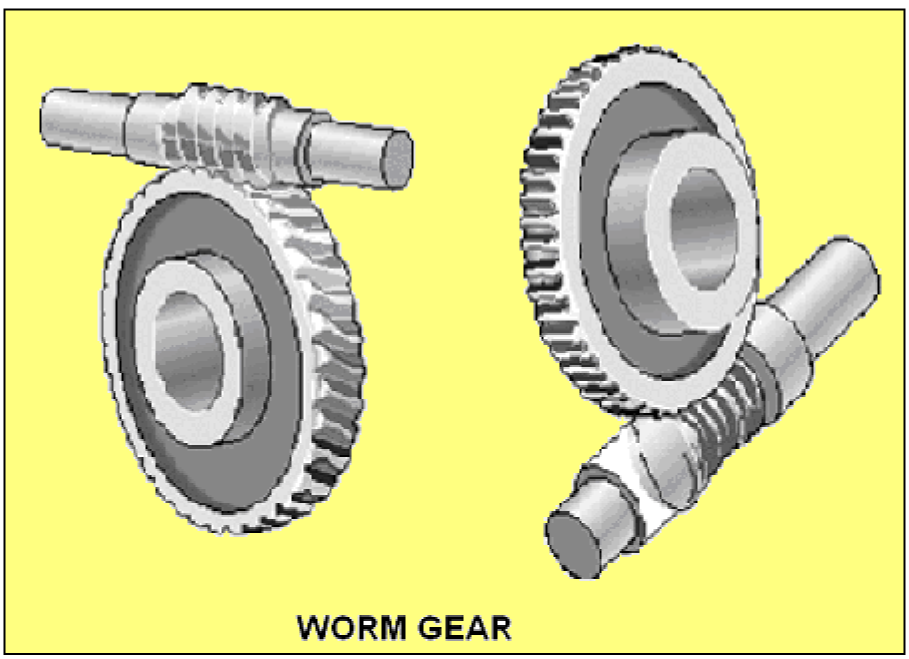
- Worm gears can provide a high angular velocity between non-intersecting shafts at right angles. They are capable of transmitting high tooth loads.
- Worm gears are used when large gear reductions are required. Worm gear has a unique property of easily turning the gear.
- They offer smoothest, quietest form of gearing. They provide high-ratio speed reduction in minimal spaces.

Types: There are three types of worm gears:

- Non-throated- a helical gear with a straight worm. Tooth contact is a single moving point on the worm drive.
- Single throated- has concave helical teeth wrap around the worm. This leads to line contact.
- Double throated- called a cone or hourglass. It has concave teeth both on the worm and helical gear

Limitations:

- The only disadvantage of worm gear is the high sliding velocities across the teeth, which results in high friction losses. Because of high friction worm gears are very inefficient.
- When used in high torque applications, the friction causes the wear on the gear teeth and erosion of restraining surface.



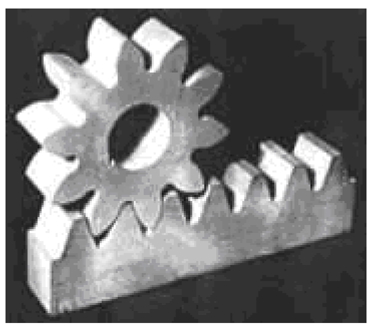
RACKS (STRAIGHT GEARS)

A rack is usually a gear without curvature (i.e. of infinite diameter) and when used with a meshing pinion enables rotary to linear movement or vice versa.

- They will mesh with pinions of the same pitch.
- This is the only gearing mechanism that converts rotational motion to translational motion. Racks are made of various materials. The commonly used materials for racks are stainless steel, brass, and plastic.

Applications:

- Efficiently transmits power. Generally offers better precision than other conversion methods.
- Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past. The steering wheel of a car rotates the gear that engages the rack. The rack slides right or left, when the gear turns, depending on the way we turn the wheel. Windshield wipers in cars are powered by a rack and pinion mechanism. They are also used in some scales to turn the dial that displays weight.



RACK & PINION

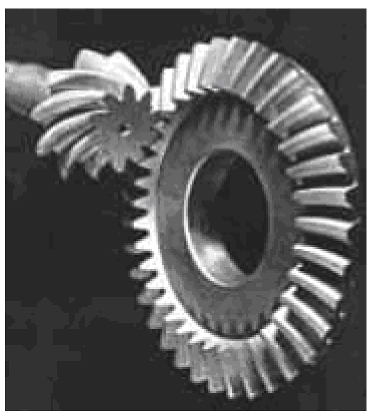
HERRINGBONE GEARS

In the case of spur gears, the tooth forces act only normal to the gear axis, whereas in case of helical gears, an additional component of force also acts along the gear axis. Its effect, however , can be eliminated by using two gears of opposite helix together or a gear may be fabricated such that half of its width is cut with helix in one direction and the other half of the teeth are cut in the opposite direction. Such a gear is called the Herringbone gear.

- They conduct power and motion between non-intersecting, parallel axis that may or may not have center groove with each group making two opposite helices. Action is equal in force and friction on both gears and all bearings.

- The most common application is in heavy machinery and power transmission. They utilize curved teeth for efficient, high capacity power transmission. This offers reduced pulsation due to which they are highly used for extrusion and polymerization.

HYPOID GEARS - One of a number of gear types for offset shafts



HYPOID GEARS

INTERNAL GEAR

Internal gears are hollow with teeth cut into the inside of the rim while the outside diameter is smooth. Gears make an internal contact with these gears.

The properties and teeth shape is similar as of external gears except that the internal gear had different addendum and dedendum values modified to prevent interference in internal meshes.

Characteristics:

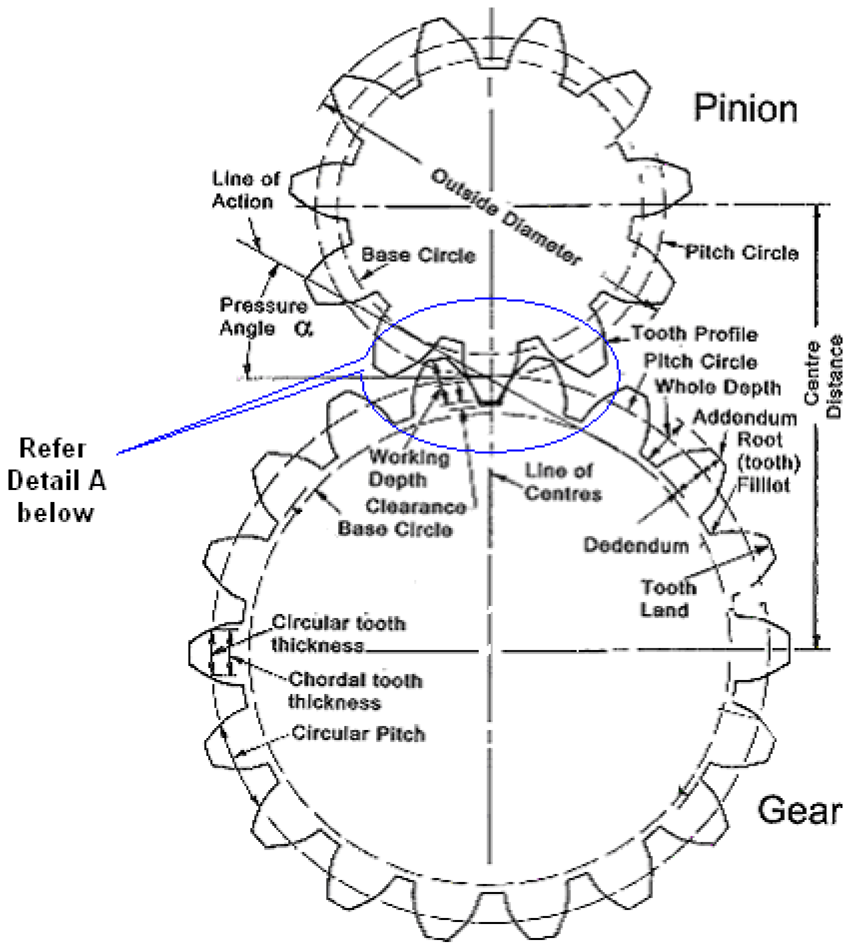
- The meshing arrangement enables a greater load carrying capacity with improved safety (since meshing teeth are enclosed) compared to equivalent external gears.
- Shaft axes remain parallel and enable a compact reduction with rotation in the same sense. Internal gears are not widely available as standard.
- Internal gear offers low sliding and high stress loading. They are used in planetary gears to produce large reduction ratios.
- When they are used with the pinion, more teeth carry the load that is evenly distributed. The even distribution decreases the pressure intensity and increases the life of the gear.
- Allows compact design since the center distance is less than for external gears.
- A high contact ratio is possible.
- Provides good surface endurance due to a convex profile surface working against a concave surface.

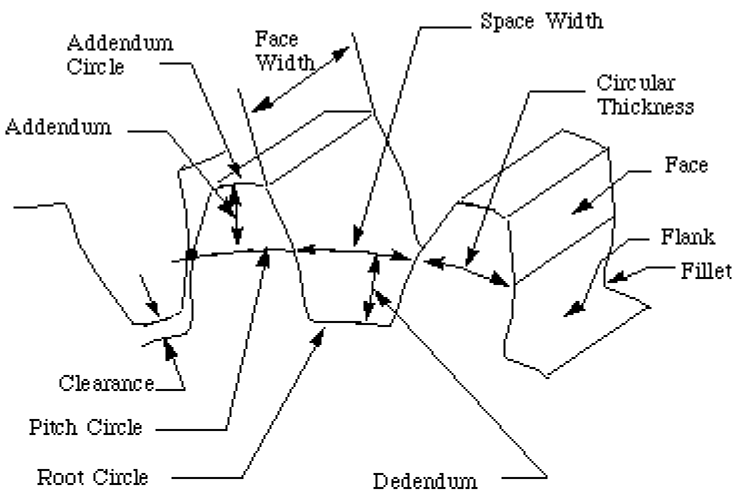
Disadvantages:

- Housing and bearing supports are more complicated, because the external gear nests within the internal gear.
- Low ratios are unsuitable and in many cases impossible because of interferences.
- Fabrication is limited to the shaper generating process, and usually special tooling is required.

TERMINOLOGY FOR GEARS

Some of the main features of spur gear teeth are illustrated in the figure below. The teeth extend from the root, or dedendum cylinder (or colloquially, "circle") to the tip, or addendum circle: both these circles can be measured. The useful portion of the tooth is the flank (or face), it is this surface which contacts the mating gear. The fillet in the root region is kinematically irrelevant since there is no contact there, but it is important insofar as fatigue is concerned. In the section below, we define many of the terms used in the analysis of spur gears. The diagram shows a spur gear but the same principles apply to spiral gears and bevel gears.





DETAIL - A

Terminology - spur gears

The essential terminology of a gear mesh is defined below:

- Base circle…..It is a circle from which involute form of gear is generated. Only the base circle on a gear is fixed and unalterable.
- Pitch circle….. It is an imaginary circle most useful in calculations. It may be noted that an infinite number of pitch circles can be chosen, each associated with its own pressure angle.
- Pitch circle diameter (D)...... It is the diameter of a circle (pitch circle) which by pure rolling action would produce the same motion as the toothed gear wheel. This is the most important diameter in gears.
- Centre distance...... The distance between the axes of two gears in mesh (or the distance between the centers of two pitch circles.
- Pitch point...... The point at which the pitch circle diameters of two gears in mesh coincide.
- Outside diameter...... The outside diameter of the gear.
- Base Circle diameter...... The diameter on which the involute teeth profile is based.
- Addendum circle…..A circle bounding the ends of the teeth, in a right section of the gear.
- Root (or dedendum) circle…..The circle bounding the spaces between the teeth, in a right section of the gear.
- Module (m)…..It is the ratio of the pitch diameter to the number of teeth i.e.

$$m = \frac{D_p}{N_p} = \frac{D_w}{N_w}$$

Where

- o Dp = Pitch diameter of pinion
- o Np = Number of teeth of pinon
- o Dw = Pitch diameter of wheel
- o Nw = Number of teeth of wheel

Alternatively module is equal *to* ***1/diameteral pitch***. The unit of the module is millimeters and is mainly used for metal gears. Below is a diagram showing the relative size of teeth machined in a rack with module ranging from module values of 0, 5 mm to 6 mm. A higher module indicates coarser tooth spacing.

- Pitch…… It is a measure of tooth spacing along the pitch circle. There are two basic forms, circular pitch and diametral pitch.

  - ➢ Circular pitch (CP)…..The spacing of gear teeth, measured on the pitch circle. The circular pitch, therefore, equals the pitch circumference divided by the number of teeth. i.e.

$$CP = \frac{\pi * D}{N} = \pi m$$

Where

CP = circular pitch

N = number of teeth

D = pitch diameter

  - ➢ Diametral pitch (DP)….. The diametral pitch is, by definition, the number of teeth divided by the pitch diameter.

$$DP = \frac{N}{D}$$

Where

- o DP = diametral pitch
- o N = number of teeth
- o D = pitch diameter

*Both the pitches are inversely related to each other and permits an easy transformation from one to the other and the product of the diametral pitch and the circular pitch equals "pi", i.e. CP * DP = π*

- Addendum...... The height of the tooth above the pitch circle diameter; also, the radial distance between the pitch circle and the addendum circle. Its value is equal to one module.
- Clearance…..This is a radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Its standard value is 0.157m.
- Dedendum...... This is a radial distance from the pitch circle to the bottom of the tooth space. Dedendum is bigger than addendum and is = Addendum + Clearance = m + 0.157m = 1.157m
- Circular thickness (also called the *tooth thickness*)….. The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line. Normally the tooth thickness = ½ * CP = ½ * π * m

  But thickness is usually reduced by certain amount to allow for some amount of backlash and owing to addendum correction.
- Back Lash...... The distance through which a gear can be rotated to bring its non working flank in contact with the teeth of mating gear.
- Pressure angle...... The angle between the line of force between meshing teeth and the tangent to the pitch circle at the point of mesh. Gears must have the <u>same pitch and the pressure angle</u> to mesh. *Larger pressure angle results in wider base and stronger teeth.* This is generally standardized at 20^{o}. Other pressure angles should be used only for special reasons and using considered judgment. The following changes result from increasing the pressure angle
  - ➢ Reduction in the danger of undercutting and interference
  - ➢ Reduction of slipping speeds
  - ➢ Increased loading capacity in contact, seizure and wear
  - ➢ Increased rigidity of the toothing
  - ➢ Increased noise and radial forces
  - ➢ Gears required having low noise levels have pressure angles 15^{o} to17.5^{o}
- Whole depth...... The total depth of the space between adjacent teeth and is equal to addendum plus dedendum, also equal to working depth plus clearance.

- Working depth…..Working depth is the depth of engagement of two gears; that is, the sum of their addendums.
- Pitch point…..The point of tangency of the pitch circles of a pair of mating gears.
- Common tangent…..The line tangent to the pitch circles at the pitch point.
- Path of contact…..The path traced by the contact point of a pair of tooth profiles.
- Tooth space….. It is the width of space between two teeth measured on the pitch circle.
- Face of tooth…. It is that part of the tooth surface which is above the pitch surface.
- Flank of the tooth…. It is that part of the tooth surface which is lying below the pitch surface.
- Face width…. It is the width of gear measured along its axis.
- Arc of approach… It is the arc measured on pitch circle from the point of beginning of contact to the pitch point.
- Arc of recess…. It is the arc measured on pitch circle from the point and position of tooth at the end of contact.
- Arc of action…. It is the sum of the arc of approach and the arc of recess.

SPUR GEAR FORMULAS

To Get...	When you know...	Formula
Diametral Pitch (DP)	Pitch Diameter (D) and the Number of Teeth (N)	$DP = \frac{N}{D}$
Diametral Pitch (DP)	Circular Pitch (CP)	$DP = \frac{\pi}{CP}$
Diametral Pitch (DP)	Outside Diameter (OD) and the Number of Teeth (N)	$DP = \frac{(N+2)}{OD}$
Diametral Pitch (DP)	Module (m)	DP = 1/m
Pitch Diameter (D)	Number of teeth (N) and the Diametral Pitch (DP)	$D = \frac{N}{DP}$
Pitch Diameter (D)	Number of teeth (N) and module (m)	$D = N * m$
Outside Diameter (OD)	Number of teeth (N) and the Diametral Pitch (DP)	$OD = \frac{(N+2)}{DP}$
Outside Diameter (OD)	Number of teeth (N) and the module	OD = (N + 2) * m

To Get...	When you know...	Formula
Number of Teeth (N)	Pitch Diameter (D) and the Diametral Pitch (DP)	N = D * DP
Addendum (a)	Diametral Pitch (DP)	$a = \frac{1}{DP}$
Addendum (a)	Circular Pitch (CP)	$a = \frac{CP}{\pi} = m$
Dedendum (d)	Whole Depth (hw) and Addendum (a)	d = hw - a
Dedendum (d)	Module (m) and clearance (Cl)	d = m + Cl (~ 1.157 *m)
Dedendum (d)	Diametral Pitch (DP)	$d = \frac{1.2}{DP} + 0.002$
Dedendum (d)	Circular Pitch (CP)	d = 0.3979 * CP
Module (m)	Pitch diameter (d) and number of teeth (N)	$m = \frac{d}{N}$
Tooth Thickness (t)	Diametral Pitch (DP)	$t = \frac{1.5708}{DP}$
Tooth Thickness (t)	Module (m)	t = π * m / 2
Working Depth (WD)	Addendum (a)	WD = 2 *a
Working Depth (WD)	Diametral Pitch (DP)	$WD = \frac{2}{DP}$
Center Distance (C)	Normal Diametral Pitch (DP) and the Number of Teeth in pinion and wheel	$C = \frac{(Np + Nw)}{2 * DP}$
Center Distance (C)	Pitch Diameters of both gears (pinion & wheel)	$C = \frac{dp + dw}{2}$
Center Distance (C)	Number of Gear Teeth (Nw), Number of Pinion Teeth (Np) and module (m)	$C = \frac{(Np + Nw) * m}{2}$
Circular Pitch (CP)	Diametral Pitch (DP)	$CP = \frac{\pi}{DP}$
Circular Pitch (CP)	Module (m)	CP = m x π
Circular Pitch (CP)	Pitch Diameter (D) and the Number of Teeth (N)	$CP = \frac{\pi * D}{N}$

To Get...	When you know...	Formula
Circular Tooth Thickness (ctt)	Circular pitch (CP)	$ctt = \frac{CP}{2}$
Clearance (Cl)	Module (m)	Cl = 0.1 * m to 0.3 *m
Whole Depth (hw) for 20 Pitch & finer	Diametral Pitch (DP)	$hw = \frac{2.2}{DP} + 0.002$
Whole Depth (hw) for Coarser than 20 Pitch	Diametral Pitch (DP)	$hw = \frac{2.157}{DP}$
Tangential force on gears (F_t)	Tooth force between contacting teeth (F) and Pressure angle (α)	$F_t = F \cos \alpha$
Separating force on gears (F_s)	Tangential force on gears (F_t)	$F_s = F_t \tan \alpha$
Torque on driver gear (T_1)	Tangential force on gears (F_t), Pitch diameter (D_1)	$T_1 = \frac{F_t D_1}{2}$
Torque on driver gear (T_2)	Tangential force on gears (F_t), Pitch diameter (D_2)	$T_2 = \frac{F_t D_2}{2}$
Linear velocity at gear circle (v)	Angular velocity (ω) and Pitch diameter (D)	$v = \frac{\omega D}{2}$
Velocity Ratio	Angular velocity of driver and driven gear (ω_1 & ω_2)	$\text{Velocity Ratio} = \frac{\omega_1}{\omega_2}$
Input Power (P_{input})	Torque (T) and angular velocity (ω)	$P_{input} = T * \omega$
Output Power (P_{output})	Efficiency (η), (Torque (T) and angular velocity (ω)	$P_{output} = \eta * T * \omega$
Gear Ratio (GR)	RPM of driver gear (n_1), RPM of driven gear (n_2) Angular velocity of driver gear (ω_1), Angular velocity of driven gear (ω_2) Number of teeth on driver gear (N_1), Number of teeth on driven gear (N_2) Torque on driver gear (T_1), Torque on	$GR = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = \frac{N_2}{N_1}$

To Get...	When you know...	Formula
	driven gear (T_2)	

(All values in inches)

CONDITIONS OF CORRECT MESHING OF GEAR PAIR

One essential condition for correct meshing of the gears is that the size of the teeth on the pinion is the same as the size of teeth on the wheel. One measure of size is the circular pitch (CP), the distance between adjacent teeth around the pitch circle thus CP = $\pi D/N$ where N is the number of teeth on a gear of pitch diameter D. The SI measure of size is the module, $m = N/\pi$ - which should not be confused with the SI abbreviation for metre. So the geometry of pinion 1 and wheel 2 must be such that :

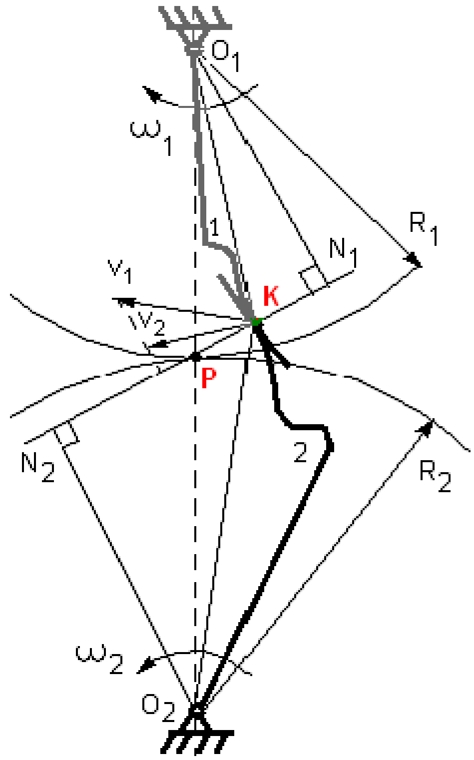
$$D_1 / N_1 = D_2 / N_2 = p/\pi = m$$

. . . . that is the module must be common to both gears.

FUNDAMENTAL LAW OF GEARING – CONJUGATE TOOTH ACTION

We have seen that one essential for correctly meshing gears is that the size of the teeth (the module) must be the same for the two gears. We now examine another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behaviour of the contacting surfaces (ie. the teeth flanks) is known as *conjugate action*.

Figure below shows two mating gears 1 and 2 which rotate about fixed centre O_1 and O_2 with angular velocities ω. The bodies touch at the contact point, K, through which the common normal N_1N_2 is drawn. N_1 and N_2 is the foot of the perpendicular from O_1 to N_1N_2 and O_2 to N_1N_2.



Although the two profiles have different velocities v_1 and v_2 at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$O_1N_1 * \omega_1 = O_2N_2 * \omega_2$

Or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N_2}{O_1N_1}$$

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point P, and

$$\Delta O_1N_1P \sim \Delta O_2N_2P$$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or ***velocity ratio***, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

The point P is called "pitch point" where the pitch circle diameters of two gears in mesh coincide and is very important to the velocity ratio. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the fundamental law of gear-tooth action.

LAW OF GEARING

A primary requirement of gear drives is the constancy of angular velocities or proportionality of position transmission. For a constant velocity ratio, the position of P should remain unchanged.

The fundamental law of gear-tooth action states that *"the common normal to the tooth profiles at their point of contact must pass through a fixed point (the pitch point) on the line of centers (to get a constant velocity ratio)".* Any two profiles which satisfy this requirement are called conjugate profiles.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the cycloidal and involute profiles. But by far the most common gear geometry which satisfies conjugacy is based on the involute, in which case both gears are similar in form, and the contact point's locus is a simple straight line - the line of action.

INVOLUTE TOOTH PROFILE

The curve most commonly used for gear-tooth profiles is the involute of a circle. This involute curve is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. We use the word *involute* because the contour of gear teeth curves inward. On an involute gear tooth, the contact point starts closer to one gear, and as the gear spins, the contact point moves away from that gear and towards the other. Involute gears have the invaluable ability of providing conjugate action when the gears' centre distance is varied either deliberately or involuntarily due to manufacturing and/or mounting errors.

The involute tooth profile is most common in use because this has many important advantages: -

- Variations in the center distance between two gears have no effect on the velocity ratio between a pair of involute gears.
- All gears are generated from a given rack (regardless of diameter and number of teeth) and so will intermesh correctly.
- It is easy to manufacture. The involute rack has straight teeth, thus the complex involute form on gear can be generated from a simple cutter.
- *The pressure angle for involute gears is always constant.* The involute system has a standard pressure angle which is either 20° or 14½° whereas on cycloidal system the pressure angle varies from zero at pitch line to a maximum at the tips of the teeth.

CYCLOIDAL TOOTH PROFILE

Cycloidal gears have a tooth shape that is cycloid generated by the rolling generating circle. They are not straight and their shape depends on the radius of the generating circle.

Cycloidal gearing requires two different curves to obtain conjugate action. Two gears are placed on either side of a roller. The roller is rolled along the outer edge of one of the gear wheels. The curve traced out from this initial point of contact is called *epicycloid*. The same roller is then rolled on the inside edge of the other gear wheel generating another curve called *hypocycloid*. These two curves will be conjugate to each other and the smaller roller disk is called the generating circle for the gear set.

Characteristics

1) Cycloidal gears are used in pairs and are set at an angle of 180 degrees to balance the load and are driven by multiple crank shafts to share the load and increase torsion rigidity. The combined tooth contact area of the two cycloidal gears and pins ensures that the load is distributed almost entirely around the pitch circle.
2) With cycloidal gearing the input and output remains in constant mesh.
3) Cycloidal gearing provides considerable latitude in selection of operating characteristics-deceleration, dwell periods, ratio of input to output motions etc.
4) In cycloidal gear if the output crank is to stop then the drive pin must be on the pitch circle of the planet gear to avoid reversing of the motion.

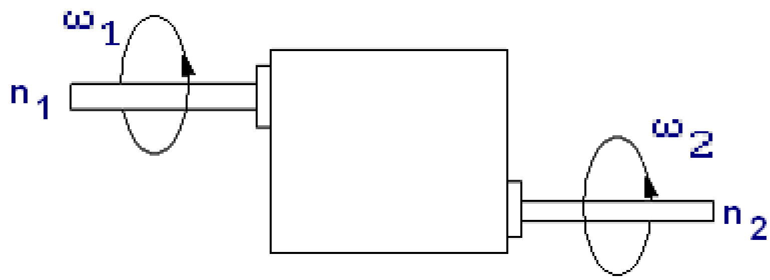
BASIC GEARBOX THEORY

Gearbox is defined as a metal casing in which a train of gears is sealed. Gearbox is also called gearhead, gear reducers or speed reducers. They are available in broad range of sizes, capacities and speed ratios. Their job is to convert the input provided by a prime mover into an output of lower RPM and correspondingly, higher torque.

Input and output configurations for gearboxes include hollow shaft or coupling or bushing. A single input shaft can drive multiple output shafts. The output shafts are usually parallel and in-line. However, some unique configurations exist that allow for offset shafts to be driven at different speeds. Some gearboxes are supplied with a reaction arm. A reaction arm prevents the reducer housing from rotating when there is no base mounts or flanges. Shaft alignment can be parallel in-line, parallel offset, right angle and non-perpendicular angled shafts.

VELOCITY RATIO AND GEAR RATIO

Consider a simple schematic of a gearbox with an input and output shaft:



Gear ratio is a number, usually expressed as a decimal fraction, representing how many turns of the input shaft cause one revolution of the output shaft. The speed ratio is more usually referred to as the gear ratio and this term is used here

$$\text{Gear Ratio} = \frac{\text{Input Speed}(n_1)}{\text{Output Speed}(n_2)} = \frac{\omega_1}{\omega_2}$$

Speed (n) is usually in revolutions per minute (RPM) but the ratio is the same whatever units of speed are used. The ω is the angular velocity.

SPEED RATIO & POWER RATIO

Gear ratio can also be defined as the relationship (ratio) between numbers of teeth on the two gear wheels.

A high gear ratio implies a high torque. If the input gear is turning faster than the output gear, the system is said to have "power ratio". If the input gear is turning slower than the output gear then the system is said to have a "speed ratio".

The special case of gear ratios is the engine speed of the car to the rotation of the drive wheels. In top gears, one turn of the engine crankshaft results in one turn of the drive wheels. Lower gears require more turns of the engine to provide single turn of the drive wheels, producing more torque at the drive wheel. For example, if the driving gear has 10 teeth and the driven gear has 20 teeth, the gear ratio is 2 to 1. Every revolution of the driving gear will cause the driven gear to revolve through only half a turn. Thus, if the engine is operating at 2,000 rpm, the speed of the driven gear will be only 1,000 rpm; the speed ratio is then 2 to 1. This arrangement doubles the torque on the shaft of the driven unit. The speed of the driven unit, however, is only half that of the engine. On the other hand, if the driving gear has 20 teeth and the driven gear has 10 teeth, the speed ratio is 1 to 2, and the speed of the driven gear is doubled. The rule applies equally well when an odd number of teeth is involved. If the ratio of the teeth is 37 to 15, the speed ratio is slightly less than 2.47 to 1. In other words, the driving gear will turn through almost two and a half revolutions while the driven gear makes one revolution.

TORQUE & EFFICIENCY

The power (in watts) transmitted by a torque T (in Nm) applied to shaft rotating at 'n rev/min' is given by 2 * π * n * T / 60. In an ideal gear box, the input and output powers are the same so

$$\frac{2\pi n_1 T_1}{60} = \frac{2\pi n_2 T_2}{60}$$

Or

$n_1 T_1 = n_2 T_2$

$$\frac{T_2}{T_1} = \frac{n_1}{n_2} = GR$$

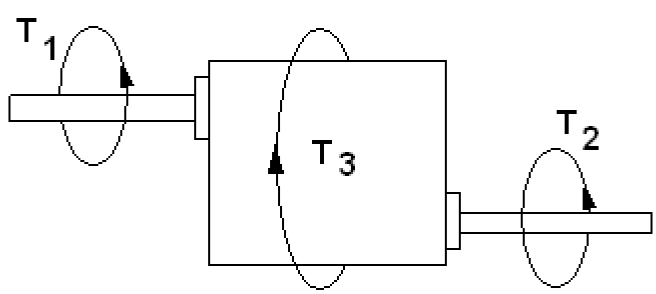
It follows that if the speed is reduced, the torque is increased and vice versa. The gear with the greater number of teeth, which will always revolve more slowly than the gear with the smaller number of teeth, will produce the greater torque. Gear trains that change speed always change torque. When speed increases, the torque decreases proportionally.

In a real gear box, power is lost through friction and the power output is smaller than the power input. The mechanical force of any driven unit will always be somewhat less than that of the driving unit due to power losses caused by such things as friction and lost motion. Therefore, out-put kW (horsepower) will always be equal to input kW (horsepower) minus any losses.

The efficiency is defined as

$$\eta = \frac{\text{Power Out}}{\text{Power In}} = \frac{2\pi n_1 T_2 \times 60}{2\pi n_2 T_1 \times 60} = \frac{n_1 T_2}{n_2 T_1}$$

Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque T3 must be applied to the body through the clamps.



The total torque must add up to zero. $T_1 + T_2 + T_3 = 0$

If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.

Example: A gear box has an input speed of 1500 RPM clockwise and an output speed of 300 RPM anticlockwise. The input power is 20 kW and the efficiency is 70%. Determine a) gear ratio, b) input torque; c) output power; d) output torque and e) the holding torque.

Solution

1) Gear Ratio

GR = Input speed / Output speed = n_1 / n_2 = 1500 / 300 = 5

2) Input Torque

Power in = 2 π n_1 T_1 / 60, therefore T_1 = 60 x Power in / (2 * π * n_1)

T_1 = 60 x 20000 / (2 * 3.14 * 1500) = 127.3 Nm (Negative clockwise)

3) Output Power

Efficiency = 0.7 = Power out / Power in, therefore Power out = 0.7 x Power in = 0.7 x 20 = 14 kW

4) Output Torque

Power out = 2 π n_2 T_2 x 60 therefore T_2 = 60 x Power out / (2 * π * n_2)

T_2 = 60 x 14000 / (2 * 3.14 * 300) = 445.6 Nm (Positive anti-clockwise)

5) Holding Torque (T_3)

We know the total torque must add up to zero $T_1 + T_2 + T_3 = 0$ or

$-127.3 + 445.6 + T_3 = 0$

T_3 = 127.3 – 445.6 = -318.3 (Anti-clockwise)

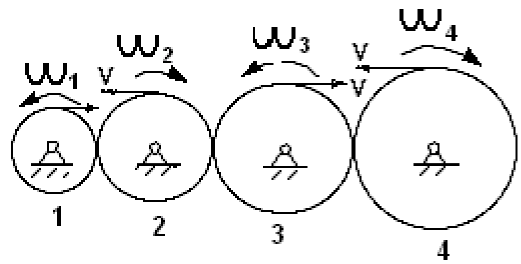
TYPES OF GEAR TRAINS

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train. The smaller gears are one-fifth of the size of the larger gear.

SIMPLE GEAR TRAIN

In a simple gear train, each shaft carries one wheel only. The figure below shows an example of simple spur gear train. The direction of rotation is reversed from one gear to another. The main limitation of a simple gear train is that the maximum speed change ratio is 10:1. For larger ratio, large size of gear trains is required; this may result in an imbalance of strength and wear capacities of the end gears.

The sprockets and chain in the bicycle is an example of simple gear train. When the paddle is pushed, the front gear is turned and that meshes with the links in the chain. The chain moves and meshes with the links in the rear gear that is attached to the rear wheel. This enables the bicycle to move.



Ordinary Gear Train

As stated above, one of the important characteristics of gear train is that the diametral pitch or the module (m) of two mating gears must be the same otherwise they would not mesh. Since the module (m) is the ratio of the pitch diameter (D) to the number of teeth (N), it follows that

$$m = \frac{D_1}{N_1} = \frac{D_2}{N_2} = \frac{D_3}{N_3} = \frac{D_4}{N_4} \quad \text{-------- Eq (1)}$$

The pitch circles contact one another at the pitch point. Since the positive drive precludes slip between the pitch cylinders, the pinion's pitch line velocity, v, must be identical to the wheel's pitch line velocity. It follows that :

$v = \omega_1 R_1 = \omega_2 R_2$; where pitch circle radius $R = D/2$ or

$$v = \frac{\omega D}{2} \quad \text{...... Eq (2)}$$

Where

- ω = Angular velocity
- D is the pitch diameter [which is proportional to number of teeth (N)]
- v is the linear velocity on the circle

Combining equation (1) and (2)

$$\frac{\omega_1 D_1}{2} = \frac{\omega_2 D_2}{2} = \frac{\omega_3 D_3}{2} = \frac{\omega_4 D_4}{2}$$

$\omega_1 * D_1 = \omega_2 * D_2 = \omega_3 * D_3 = \omega_4 * D_4$

Substituting value of D from equation (1)

$\omega_1 * m * N_1 = \omega_2 * m * N_2 = \omega_3 * m * N_3 = \omega_4 * m * N_4$

$\omega_1 * N_1 = \omega_2 * N_2 = \omega_3 * N_3 = \omega_4 * N_4$

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} \quad \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} \quad \frac{\omega_3}{\omega_4} = \frac{N_4}{N_3}$$

These equations can be combined to give the velocity ratio of the first gear in the train to the last gear:

$$\frac{\omega_1}{\omega_4} = \frac{N_2 N_3 N_4}{N_1 N_2 N_3} = \frac{N_4}{N_1}$$

In practice, rotational speed is described by n (rev/min) rather than by ω (rad/s) therefore

$$\frac{n_1}{n_4} = \frac{N_2 N_3 N_4}{N_1 N_2 N_3} = \frac{N_4}{N_1}$$

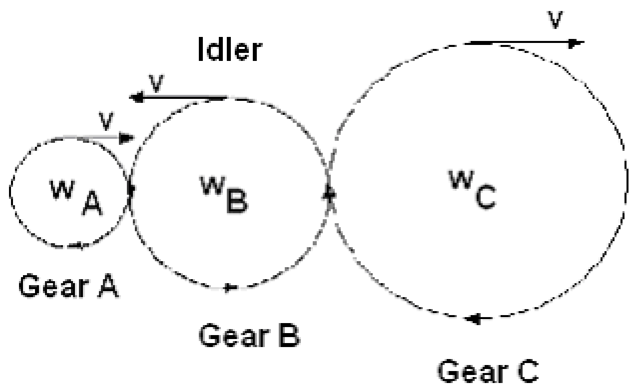
Thus, we infer that the *velocity ratio* of a pair of gears is the inverse ratio of their *number of teeth*.

It may be noted that the ratio of the speeds of gears 1 and 4 is equal to ratio of the numbers of teeth on the two gears 4 and 1, the intermediate gears having no effect. Note the direction of rotation of input and output gears. *The only way that the input and output shafts of a gear pair can be made to rotate in the same sense is by interposition of an odd number of intermediate gears.*

Note:

- The tooth number in the numerator is those of the driven gears, and the tooth numbers in the denominator belong to the driver gears.
- Gear 2 and 3 both drive and are, in turn, driven. Thus, they are called **idler gears**. Since their tooth numbers cancel, it has *no affect on the gear ratio* and the only function of the idler gear is to *change the direction of rotation*. Note the directional arrows in the figure. Idler gears can also constitute a saving of space and money (If gear 1 and 4 meshes directly across a long center distance, their *pitch circle* will be much larger.)
- There are two ways to determine the direction of the rotary direction. The first way is to label arrows for each gear as in figure above. The second way is to multiple m^{th} power of "*-1*" to the general velocity ratio. Where *m* is the number of pairs of *external contact* gears (*internal contact* gear pairs do not change the rotary direction). However, the second method cannot be applied to the spatial gear trains.

Example: A simple train has 3 gears. Gear A is the input and has 50 teeth. Gear C is the output and has 150 teeth. Gear A rotates at 1500 RPM clockwise. Calculate the gear ratio and the output speed. Also if the input torque on A is 12 Nm and the efficiency is 75%, calculate the output power and the holding torque.



Solution

1) Gear Ratio

$$\frac{n_A}{n_C} = \frac{N_C}{N_A} = \frac{150}{50} = 3$$

$$n_C = \frac{n_A}{3} = \frac{1500}{3} = 500 \text{ RPM (anti clockwise)}$$

2) Output Power

P (input) = $2 * \pi * n_A * T_A / 60 = 2 * \pi * 1500 * 12 / 60 = 1885$ W (note that $T_A = 12$ Nm)

P (output) = P (input) x efficiency = 1885 x 0.75 = 1413.7 W

3) Holding Torque

$T_C = 60 *$ P (output) / $(2 * \pi * n_C) = 60 * 1413.7 / (2 * 3.14 * 500) = 27$ Nm

$T_A + T_C + T_{HOLD} = 0$

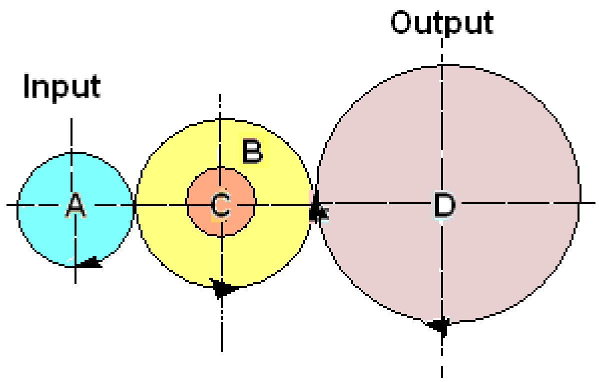
$12 + 27 + T_{HOLD} = 0$

$T_{HOLD} = -39$ Nm (clockwise)

COMPOUND GEAR TRAIN

In a compound train, each shaft, except the first and last, carries two wheels, one of which receives its motion from first wheel and the other transmits motion to a driven wheel. Figure below shows an example of compound train. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.

For large velocities, compound arrangement is preferred. Two keys are keyed to a single shaft. A double reduction train can be arranged to have its input and output shafts in a line, by choosing equal center distance for gears and pinions.



COMPOUND GEAR TRAIN

The velocity of each tooth on A and B are the same so $\omega_A * N_A = \omega_B * N_B$ as they are simple gears. Likewise for C ad D, $\omega_C * N_C = \omega_D * N_D$. (Where ω is the angular velocity and N is the number of teeth on any gear)

$$\frac{\omega_A}{N_B} = \frac{\omega_B}{N_A} \quad \text{and} \quad \frac{\omega_C}{N_D} = \frac{\omega_D}{N_C}$$

$$\omega_A = \frac{\omega_B N_B}{N_A} \quad \text{and} \quad \omega_C = \frac{\omega_D N_D}{N_C}$$

$$\omega_A \omega_C = \frac{\omega_B N_B}{N_A} \times \frac{\omega_D N_D}{N_C} = \frac{N_B N_D}{N_A N_C} \times \omega_B \omega_D$$

$$\frac{\omega_A \omega_C}{\omega_B \omega_D} = \frac{N_B N_D}{N_A N_C}$$

Since gears B and C are on the same shaft, $\omega_B = \omega_C$

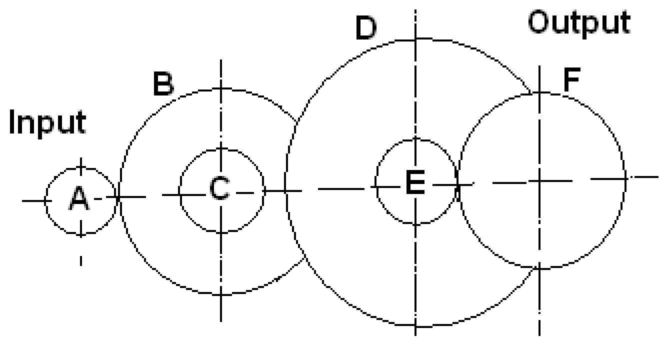
$$\frac{\omega_A}{\omega_D} = \frac{N_B N_D}{N_A N_C} = GR$$

Since ω = 2 π n, then the gear ratio may be written in terms of RPM ratio

$$\frac{n_{Input}}{n_{Output}} = \frac{N_B N_D}{N_A N_C} = GR$$

Gears B and D are the driven gears. Gears A and C are the driver gears. It follows that gear ratio is the ratio of product of driven teeth to product of driving teeth. *Thus, a large gear ratio (or speed ratio) can be obtained with wheels of smaller diameters.* This rule applies regardless of how many pairs of gears there are.

Example: Calculate the gear ratio for the compound chain shown below. If the input gear rotates clockwise, in which direction does the output rotate?



Gear A has 20 teeth

Gear B has 100 teeth

Gear C has 40 teeth

Gear D has 100 teeth

Gear E has 10 teeth

Gear F has 100 teeth

Solution

The driving teeth are A, C and E

The driven teeth are B, D and F

Gear ratio = (100 x 100 x 100) / (20 x 40 x 10) = 125

Alternatively we can say there are three simple gear trains

First gear GR = 100 / 20 = 5

Second chain GR = 100 / 40 = 2.5

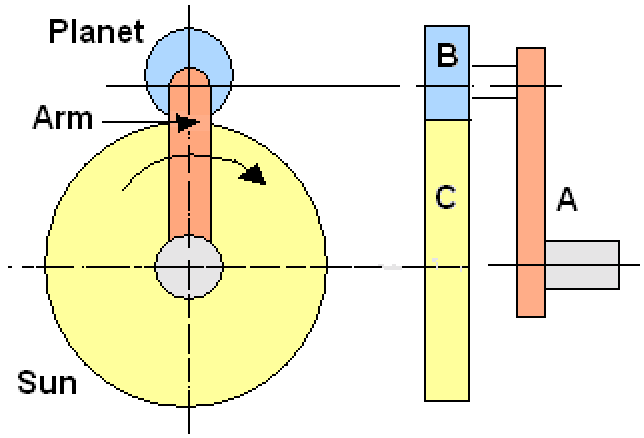
Third chain GR = 100 / 10 = 10

The overall ratio = 5 x 2.5 x 10 = 125

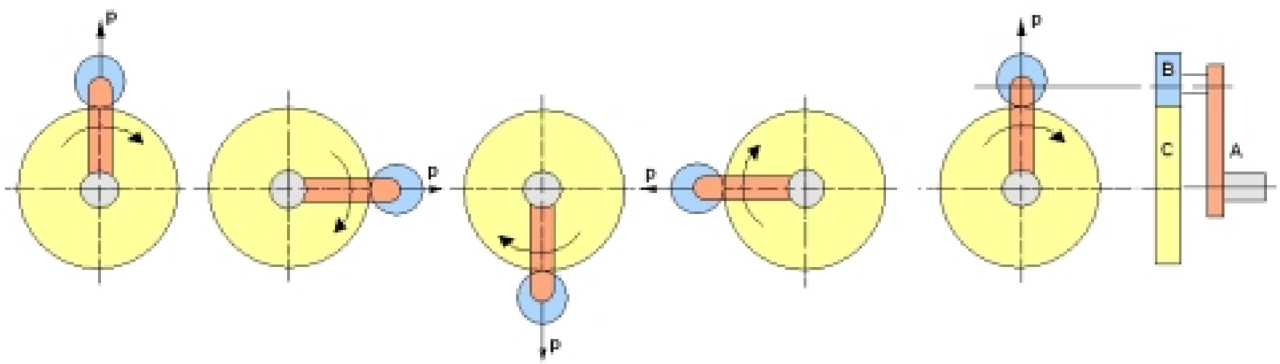
Each chain reverses the direction of rotation so if A is clockwise, B and C rotate anti-clockwise so D and E rotate clockwise. The output gear F hence rotates anti-clockwise.

EPICYCLIC GEAR TRAIN

In epicyclic gear train, the axis of rotation of one or more of the wheels is carried on an arm which is free to revolve about the axis of rotation of one of the other wheels in the train. The diagram shows a gear B on the end of an arm A. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.



Now let's see what happens when the planet gear orbits the sun gear.



Observe point p and you will see that gear B also revolves once on its own axis. Any object orbiting around a centre must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear C is rotated once. B spins about its own center and the number of revolutions it makes is the ratio N_C/ N_B. B will rotate by this number for every complete revolution of C.

Now consider that C is unable to rotate and the arm A is revolved once. Gear B will revolve 1 + (N_C / N_B) because of the orbit. It is the extra rotation that causes confusion. One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that if fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Suppose gear C is fixed and the arm A makes one revolution. Determine how many revolutions the planet gear B makes.

Step 1 is to revolve everything once about the centre.

Step 2 identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of B.

Step 3 is simply add them up and we find the total revs of C is zero and for the arm is 1.

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	$+ N_C / N_B$	-1
3	Add	1	$1 + N_C / N_B$	0

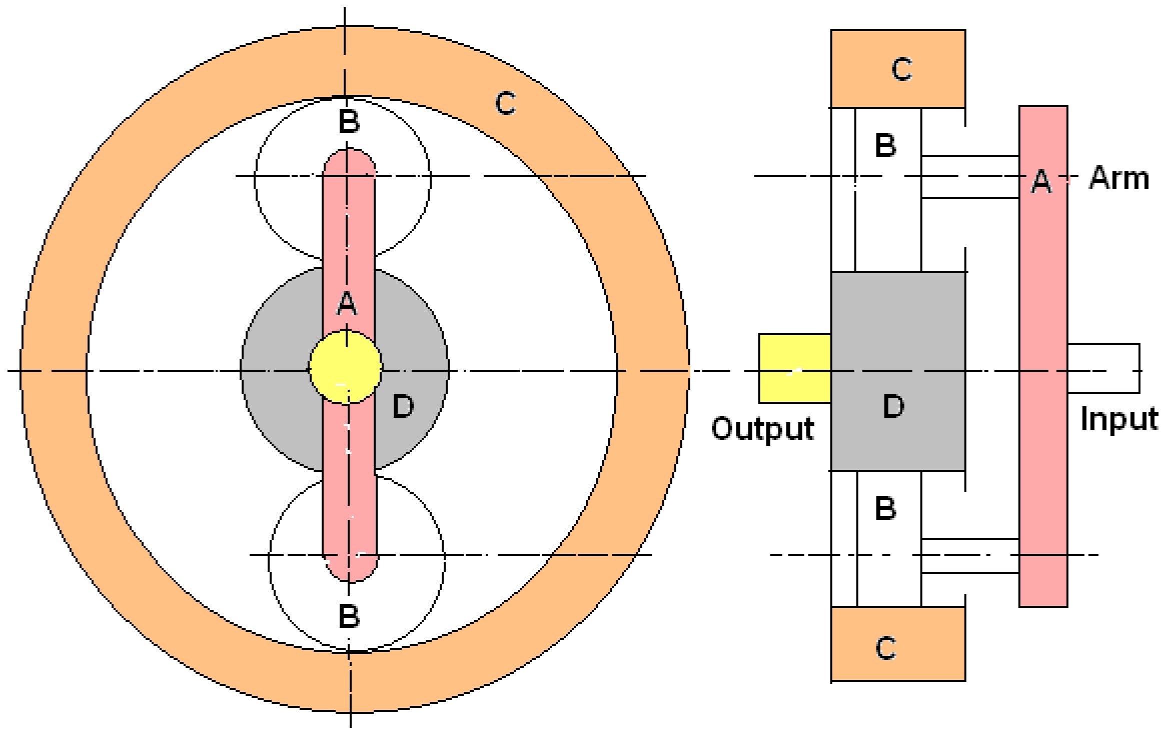
The number of revolutions made by B is $(1+ t_C /t_B)$. Note that if C revolves -1, then the direction of B is opposite so $+ t_C / t_B$

Example: A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

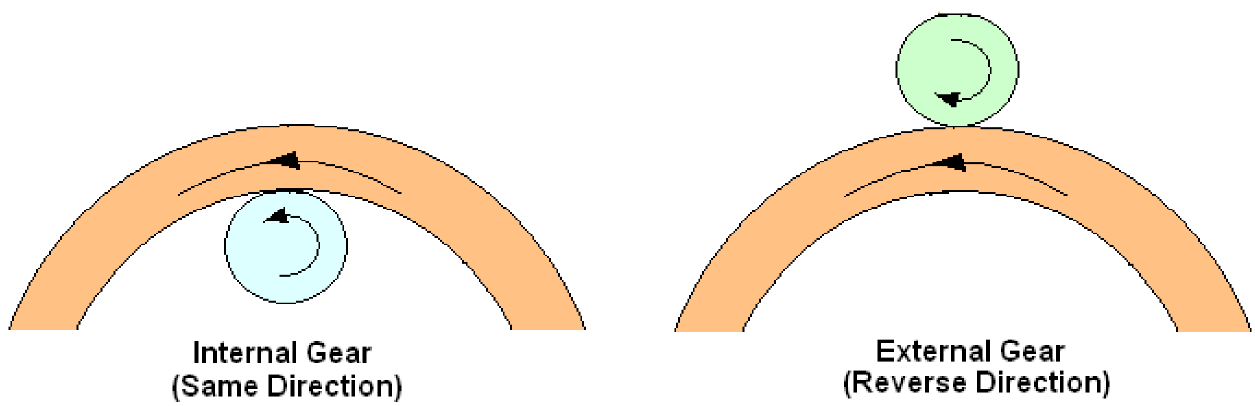
Solution

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	+ 100 / 50	-1
3	Add	1	3	0

The design so far considered has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done several ways.

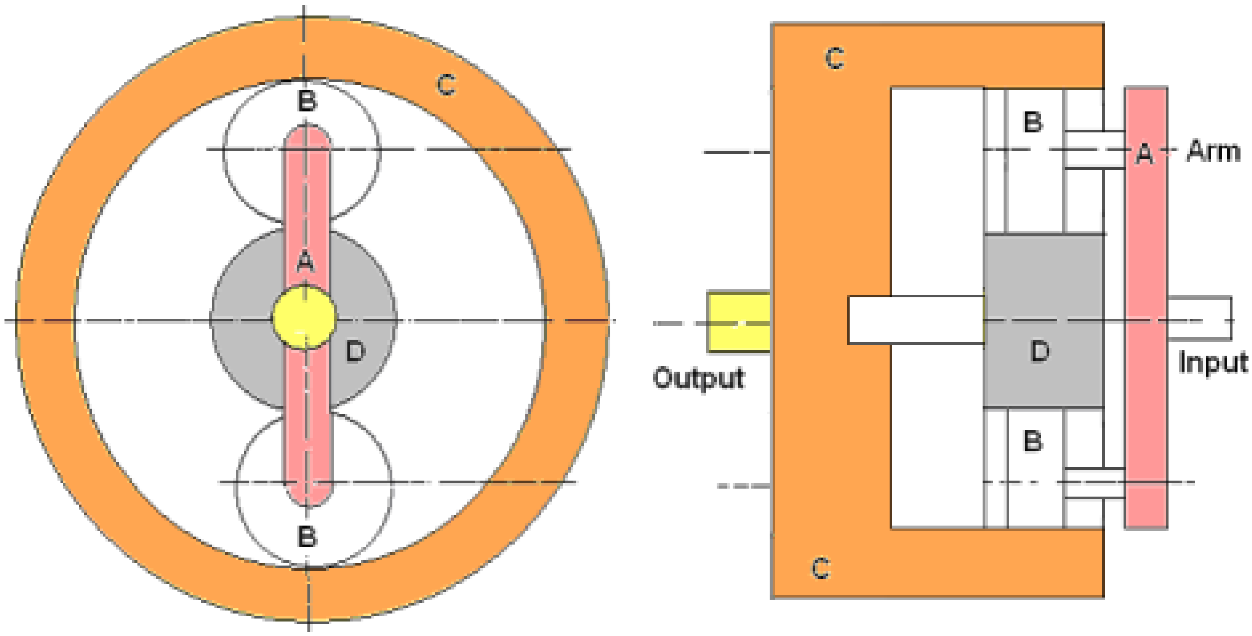


The arm is the input and gear D is the output. Gear C is a fixed internal gear and is normally part of the outer casing of the gear box. There are normally four planet gears and the arm takes the form of a cage carrying the shafts of the planet gears. Note that the planet gear and internal gear both rotate in the same direction.



Method 2

In this case the sun gear D is fixed and the internal gear C is made into the output.



Example: An epicyclic gear box has a fixed sun gear D and the internal gear C is the output with 300 teeth. The planet gears B have 30 teeth. The input is the arm /cage A. Calculate the number of teeth on the sun gear and the ratio of the gear box.

Solution

Nc = N_D + 2 N_B

300 = N_D + 2 x 30

N_D = 300 – 60 = 240

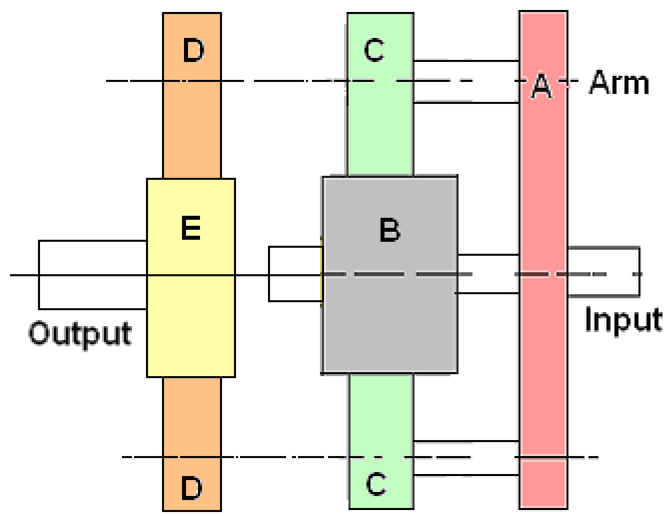
Identify that gear D is fixed and the arm must do one revolution so it must be D that is rotated back one revolution holding he arm stationary.

Step	Action	A	B	C	D
1	Revolve all once	1	1	1	1
2	Revolve D by -1 rev	0	240 / 30	240/300	-1
3	Add	1	9	1.8	0

The ratio A/C is then 1: 1.8 and this is the gear ratio. Note that the solution would be the same if the input and output are reversed but the ratio would be 1.8.

Method 3

In this design a compound gear C and D is introduced. Gear B is fixed and gears C rotate upon it and around it. Gears C are rigidity attached to gears D and they all rotate at the same speed. Gears D mesh with the output gear E.



Example: An epicylic gear box is shown above. Gear C has 100 teeth, B has 50, D has 50 and E has 100. Calculate the ratio of the gear box.

Solution

Identify that gear B is fixed and that A must do one revolution so it must be B that is rotated back one revolution holding A stationary.

Step	Action	A	B	C/D	E
1	Revolve all once	1	1	1	1
2	Revolve B by -1 rev	0	-1	½	-¼
3	Add	1	0	1½	¾

The ratio A/E is then ¾:1 or 3:4

Note that the input and output may be reversed but the solution would be the same with a ratio of 4:3 instead of 3:4.

TYPES OF GEARBOX

There are two types of gearboxes:

Automatic gearbox: Several factors have contributed to the development of automatic gearbox. Firstly, the advent of electronics in the nineties and secondly, the wish of having more gear speed. It is used for power transmission and offers automatic gearshift. Automatic gearboxes are easy and pleasurable to drive. The only thing to be done after engaging a gear is to press the accelerator to go and press the brake to stop. The automatic gearbox relies on hydraulic fluid pressure to shift the gears up and down. This fluid needs to be checked regularly.

Manual gearbox: Manual gearbox has widely spread into the market but Europeans still remain faithful to manual gearbox. The manual gearbox has been as old as the car itself. These gearboxes use conventional clutch that is activated each time a gear is selected by an electronically controlled motor. This then disengages the clutch, the gear is shifted and the clutch engages once more. It all happens within a second and the system even allows for the car to come to a stop whilst still in gear. The manual gearbox is virtually maintenance free except for the checking of the oil level occasionally.

ACCELERATION OF GEAR TRAINS

We have learnt that the linear velocity (v) of any point on the circle must be the same for all the gears, otherwise they would be slipping,

$$v = \frac{\omega\ D}{2}$$

Where

- ω = Angular velocity
- D is the pitch diameter which is proportional to number of teeth (N)
- v is the linear velocity on the circle

Also it’s not the linear velocity but also the acceleration that must be same. The acceleration (a) of a point on the circle is given by m/s^2 and is related to the angular acceleration ά by

$$a = \frac{\propto\ D}{2}$$

Where

- α = Angular acceleration
- D is the pitch diameter which is proportional to number of teeth (N)
- a is the acceleration on the circle

D is pitch diameter, proportional to the number of teeth (N). Since a is the same for all then $\alpha_A N_A = \alpha_B N_B = \alpha_C N_C$

All bodies have mass (inertia) and so a force is needed to change their motion. In the case of a wheel, torque is needed to produce changes in the angular speed and Newton’s law of motion for a wheel is

$Ti = I * \alpha$

Where

- I is the moment of inertia of the wheel
- α is the angular acceleration

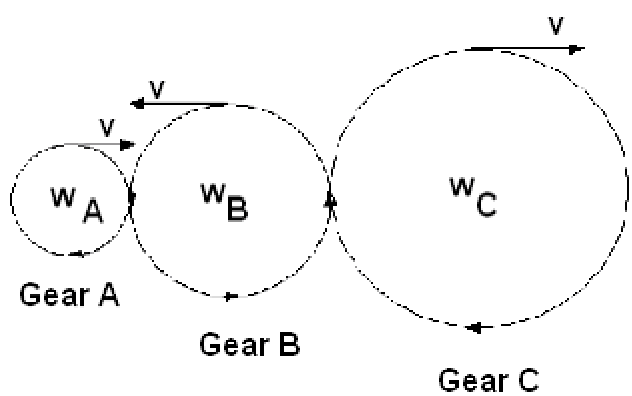
The moment of inertia is found from $I = m * k^2$

Where

- m = mass
- k is the radius of gyration of the wheel (the effective radius of the rotating mass)

In real gear trains, friction and other loads placed on the gears produce extra torque which must be added to the inertia torque. *Note the inertia torque is only produced when there are changes to the motion and not when running at constant speed.* We will only consider a simple gear train here.

Simple Gear Train



Consider simple gear train again. The power produced by a wheel is ω * T.

If a torque T_C exists on the shaft of gear C, it must have originated from a torque on the shaft of the input gear A. Assuming no energy loss; we may determine T_A by equating input and output power.

$\omega_A * T_A = \omega_B * T_B = \omega_C * T_C$

$$T_A = \frac{T_C * \omega_C}{\omega_A} = \frac{T_C * N_A}{N_C} \quad \text{...... (since } \frac{\omega_C}{\omega_A} = \frac{N_A}{N_C} \text{)}$$

Similarly if a torque exists on the shaft of gear B, the torque resulting on gear A is

$$T_A = \frac{T_B * N_A}{N_B}$$

Consider that the torque is due to acceleration of the gear train (inertia torques).

Torque required accelerating gear A: $T_A = I_A * \alpha_A$

Torque required accelerating gear B: $T_B = I_B * \alpha_B$

Torque required accelerating gear C: $T_C = I_C * \alpha_C$

All these torques must be provided by gear A. These are found by use of equation (1). The total torque on A is

$$T_A = I_A * \alpha_A + I_B \alpha_B \left(\frac{N_A}{N_B}\right) + I_C \alpha_C \left(\frac{N_A}{N_C}\right)$$

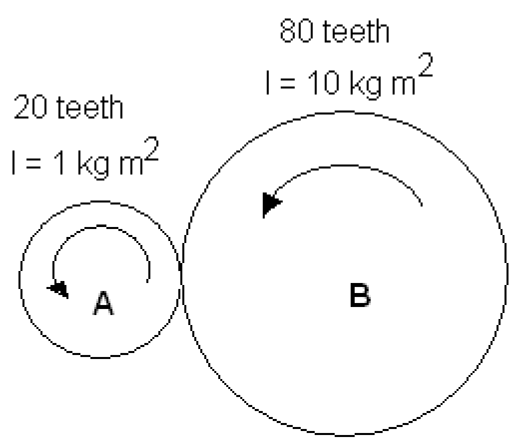
Since the accelerations are related by $\alpha_A * N_A = \alpha_B * N_B = \alpha_C * N_C$, we may convert all the accelerations into α_A. The torque becomes

$$T_A = I_A * \alpha_A + I_B \alpha_A \left(\frac{N_A}{N_B}\right)^2 + I_C \alpha_A \left(\frac{N_A}{N_C}\right)^2$$

$$T_A = \alpha_A \left[I_A + I_B \left(\frac{N_A}{N_B}\right)^2 + I_C \left(\frac{N_A}{N_C}\right)^2 \right]$$

The expression $\left[I_A + I_B \left(\frac{N_A}{N_B}\right)^2 + I_C \left(\frac{N_A}{N_C}\right)^2 \right]$ is called the effective moment of inertia.

Example: Calculate the input torque required to accelerate the gear A at 6.67 rad/s^2.



Solution

$I_{(Effective)} = [I_A + I_B * (N_A / N_B)^2] = 1 + 10\,(20/80)^2 = 1.625$

$T_A = \alpha_A * I = 6.67 \times 1.625 = 10.8$ Nm

GEAR MATERIALS

Gears may break by fatigue failure due to the cyclical rubbing together of gear teeth in mesh. The friction heat may cause pitting or wear at the tooth face. When selecting a suitable material, properties such as resistance to wear and good fatigue strength as well as a low coefficient of friction are, therefore, desirable. Alloy steels are most commonly used in industrial field. They offer high strength and a wide range of heat treatment properties. The material composition below indicates how properties vary with commonly used alloy materials:

1) Nickel - Increases hardness and strength.

2) Chromium - Increases hardness and strength but the loss of ductility is greater. It refines the grain and imparts a greater depth of hardness. It has high degree of wear resistance.

3) Manganese - It gives greater strength and a high degree of toughness than chromium.

4) Vanadium - The hardness penetration is greatest. The loss of ductility is also more than any other alloys.

5) Molybdenum - Increases strength without affecting the ductility.

6) Chrome - Nickel Steels - The combination of the two alloying elements chromium and nickel adds the beneficial qualities of both.

Mild steel is a poor material for gears as it has poor resistance to surface loading. The carbon content for unhardened gears is generally 0.4 %(min) with 0.55 %(min) carbon for the pinions.

Stainless steel gear may be stainless steel (austenitic), which is non-magnetic and has good corrosion resistance or they may be of stainless steel (martensitic) that can be easily hardened by heat, is magnetic and have reasonable corrosion resistance. Stainless steel (austenitic) can be used where low power ratings are there and the other stainless steel is used where low to medium power ratings is there.

Material	Notes	applications
Ferrous metals		
Cast Iron	Low Cost easy to machine with high damping	Large moderate power, commercial gears
Cast Steels	Low cost, reasonable strength	Power gears with medium rating to commercial quality
Plain-Carbon Steels	Good machining, can be heat treated	Power gears with medium rating to commercial/medium quality
Alloy Steels	Heat Treatable to provide highest strength and durability	Highest power requirement. For precision and high precisiont

Material	Notes	applications
Ferrous metals		
Stainless Steels (Aust)	Good corrosion resistance. Non-magnetic	Corrosion resistance with low power ratings. Up to precision quality
Stainless Steels (Mart)	Harden able, Reasonable corrosion resistance, magnetic	Low to medium power ratings Up to high precision levels of quality

Material	Notes	applications
Non-Ferrous metals		
Aluminum alloys	Light weight, non-corrosive and good machinability	Light duty instrument gears up to high precision quality
Brass alloys	Low cost, non-corrosive, excellent machinability	low cost commercial quality gears. Quality up to medium precision
Bronze alloys	Excellent machinability, low friction and good compatibility with steel	For use with steel power gears. Quality up to high precision
Magnesium alloys	Light weight with poor corrosion resistance	Light weight low load gears. Quality up to medium precision
Nickel alloys	Low coefficient of thermal expansion. Poor machinability	Special gears for thermal applications to commercial quality
Titanium alloys	High strength, for low weight, good corrosion resistance	Special light weight high strength gears to medium precision
Di-cast alloys	Low cost with low precision and strength	High production, low quality gears to commercial quality
Sintered powder alloys	Low cost, low quality, moderate strength	High production, low quality to moderate commercial quality

Material	Notes	applications
Non metals		
Acetal (Delrin	Wear resistant, low water absorbtion	Long life , low load bearings to commercial quality

Material	Notes	applications
Non metals		
Phenolic laminates	Low cost, low quality, moderate strength	High production, low quality to moderate commercial quality
Nylons	No lubrication, no lubricant, absorbs water	Long life at low loads to commercial quality
PTFE	Low friction and no lubrication	Special low friction gears to commercial quality

It is not essential for both pinion and wheel gears to be of the same material. As the smaller gear will have to rotate more frequently than the larger gear it is more prone to wear and fatigue. It is common, therefore, to choose a material with improved properties for the pinion to give a gear pair with a near matching strength and durability.

GEAR LUBRICATION

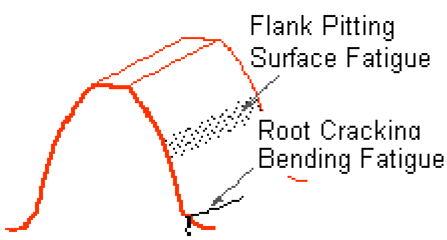
Tooth losses occur due to the rubbing together of mating gear teeth and the amount of heat which is generated is a measure of the loss in efficiency. Losses are generally greater with large tooth loads and with increased friction surfaces. The use of a lubricant and using gears made from low friction material with teeth having a smooth surface finish helps keep friction losses to a minimum. A lubricant has the added advantage, also, of carrying away any heat that is generated from the vicinity of the teeth.

The choice of lubricant depends on operating conditions.

- At peripheral speeds up to 18 m/s it is preferable to use a lubricant with a low viscosity to avoid excessive churning of the fluid and to facilitate splash lubrication.
- For very high gear forces lubricants with a greater viscosity are used and for faster gear speeds a pressurized feed system may be necessary.

GEAR FAILURE – RELIABILITY ANALYSIS

The gears are subjected to high stresses, which occur where the teeth contact one another. Bending stresses occur at the root of the lower cantilevered tooth whose fillet radius is large. Much higher bending stresses occur at the root of the upper tooth whose fillet radius is small.



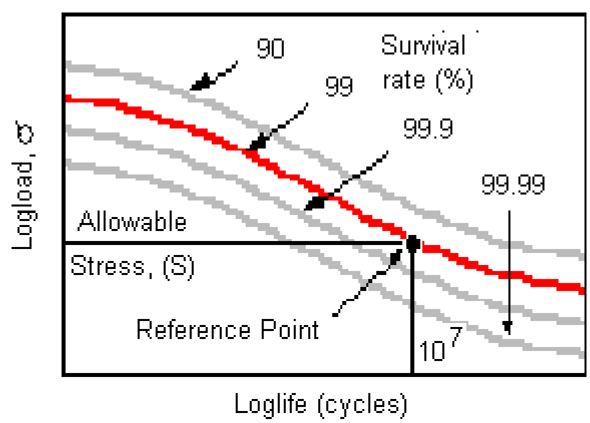
These stresses are alternating since a particular tooth is loaded only briefly during one rotation of the gear. Gear failure is therefore very much a case of fatigue, though a one-off static overload obviously may cause failure if sufficiently large. Apart from one-off overloads, there are three common modes of tooth failure

1) ***Bending*** fatigue leading to root cracking,

2) Surface contact fatigue leading to flank ***pitting,*** and

3) Lubrication breakdown leading to ***scuffing.***

The fatigue is the most prevalent failure mode and therefore various standards such as the AGMA or ISO employ a reliability approach rather than a safety factor approach in assessing a tooth's tolerance of damage.In a reliability analysis, knowledge of a component's load-life (S-N) relationship enables a load to be considered from the point of view of its effect on component life rather than whether it leads to total failure or to total non-failure. A smaller load increases life, a larger load reduces life; whether the safety factor is greater or less than one is irrelevant - indeed the whole concept of safety factor is inappropriate in the context of reliability. The load-life diagram for a particular material and given type of loading is generally of the form illustrated below, with curves corresponding to different survival rates' being approximately parallel to one another - the 99% survival curve for example implies that one sample in a hundred fails to reach the life given by the curve for a particular load.



The reliability curves are for steel materials (carbon steels induction- or through-hardened, nitrided alloy steels etc). Rather than providing the complete load-life diagram for each steel, the AGMA chooses a reference point on the curve corresponding to 99% survival rate after 10^7 unidirectional loading cycles, and cites the corresponding allowable stress, S, as a representative property of the steel. So, for any given load (σ) the life and survival rate (reliability) may be correlated through: $\sigma = S \ast$ life factor (K_L or C_L) / reliability factor (K_R) where:

The *reliability factor,* K_R, caters for survival rates other than 99%. Since the survival contours are essentially parallel to one another on a logarithmic scale, then simple multiplying factors enable load correlation, as tabled :-

RELIABILITY FACTOR	% survival	K_R
fewer than one failure in 10,000	99.99	1.50
fewer than one failure in 1,000	99.9	1.25
fewer than one failure in 100	99	1.00
fewer than one failure in 10	90	0.85

The *life factor* (K_L for bending, C_L for pitting) caters for lives other than 10^7 cycles. Since the load-life diagrams for all the steels considered are of the same shape essentially, normalising by the allowable stress will result in a unique K_L (or C_L) -versus- life curve for all steels.

SELECTION OF GEAR

Gears can either be obtained as standard components from a manufacturer's catalogue or alternatively specially designed and manufactured. Smaller sized gears, especially instrument gears, tend to be more readily available from catalogues and larger, less used gear types tend to be produced as specials; usage as claimed by leading manufacturers is approximately equally divided.

Gear catalogues tend to display only geometric and materials data of stock gears rather than specific operational information. This is because functional behavior will vary with an application and it is not feasible to give comprehensive data covering all operational conditions within a catalogue. To select gears from a stock gear catalogue or do first approximations for a gear design select the gear material and obtain a safe working stress e.g. Yield stress / Factor of Safety /Safe fatigue stress

- Determine the input speed, output speed, ratio, torque to be transmitted
- Select materials for the gears (pinion is more highly loaded than gear)
- Determine safe working stresses (UTS /factor of safety or yield stress/factor of safety or fatigue strength / factor of safety)
- Determine Allowable endurance Stress S_e
- Select a module value and determine the resulting geometry of the gear
- Use the Lewis formula and the endurance formula to establish the resulting face width
- If the gear proportions are reasonable then - proceed to more detailed evaluations
- If the resulting face width is excessive - change the module or material or both and start again

The gear face width should be selected in the range 9-15 x module or for straight spur gears-up to 60% of the pinion diameter.

MAXIMUM STATIC TORQUE & TOOTH BENDING STRESS

Gear analysis or design usually starts with a known time-averaged (ie. steady) power transfer. In fine-pitch gearing applications, gear trains are sometimes subject to high static loads. It is extremely important that the gears be capable of with standing this maximum static torque. For reliability analysis, the maximum static torque capacity for gear is given by equation:

$$T_s = K_a * \frac{F_t * DP}{2}$$

Where

1) DP is the diametral pitch given by equation

$$\text{Diameteral Pitch (DP)} = \frac{\text{Number of Teeth (N)}}{\text{Pitch Diameter (D)}} = \frac{1}{\text{Module (m)}}$$

2) F_t is the tangential force that may be determined from the Lewis formula is used for determining the tangential force (F_t) which may safely be applied to spur gear teeth. This formula considers the gear as a cantilever beam with the full load applied to one tooth. It should be remembered that more than one tooth is actually in contact during engagement and therefore the load is partially shared with another pair of teeth. This property is called the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. *The higher the contact ratio the more the load is shared between teeth.* <u>It is good practice to maintain a contact ratio of 1.3 to 1.8.</u> Under no circumstances should the ratio drop below 1.1. A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact.

3) Ka is an application factor to allow for the non-uniformity of input and/or output torque inherent in the machinery connected to the gears.

APPLICATION FACTOR K_a FOR REDUCTION GEARS

Driven Machinery		Driving Machinery		
		Uniform Electric motor, Steam turbine Gas turbine	Light Shocks Multi-cylinder combustion engine	Heavy Shocks Single cylinder combustion engine
Uniform	Generator, Belt conveyor, Light Elevator, Electric hoist, Machine tool feed drive, Ventilator, Turbo-blower, Turbo-compressor, Mixer (constant density)	1	1.25	1.5
Medium Shocks	Machine tool main drive, Heavy elevator, Crane turning gears, Mine ventilator, Mixer (variable density), Multi-cylinder, Piston pump, Feed pump	1.25	1.5	1.75
Heavy Shocks	Press, Shear, Rolling mill drive, Heavy centrifuge, Heavy feed pump, Pug mill, Power shovel, Rotary drilling apparatus, Briquette press	1.75	2	2.25

LEWIS EQUATION

According to Lewis equation for design of gear tooth, tangential force on gear tooth is

$F_t = \sigma * w * m * Y$

- F_t = Maximum safe tangential force on tooth at pitch diameter, Kg
- σ = Permissible stress in MPa
- w = Face width (mm)
- m = Module (mm)
- Y = Lewis Form Factor

When a gear wheel is rotating the gear teeth come into contact with some degree of impact. To allow for this a velocity factor is introduced into the equation. This is given by the Barth equation for milled profile gears.

$F_t = \sigma * w * m * Y * K_v$

Where

K_v = Velocity factor and is given by equation

$K_v = 6 / (6 + V)$

V = the pitch line velocity and is given by D * ω / 2.

Note: *This factor is different for different gear conditions i.e. $K_v = (3.05 + V)/3.05$ for cast iron, cast profile gears.*

Y the Lewis form factor taken from the following table:

Number of teeth	Lewis form factor	Number of teeth	Lewis form factor	Number of teeth	Lewis form factor	Number of teeth	Lewis form factor	Number of teeth	Lewis form factor
12	0.230	17	0.285	22	0.320	34	0.367	75	0.423
13	0.243	18	0.293	24	0.331	38	0.377	100	0.436
14	0.255	19	0.301	26	0.340	45	0.391	150	0.449
15	0.266	20	0.308	28	0.348	50	0.399	300	0.464
16	0.276	21	0.314	30	0.355	60	0.410	Rack	0.479

Choose the number of teeth for the gears

A single spur gear is generally selected to have a ratio range of between 1:1 and 1:6 with a pitch line velocity up to 25 m/s. The spur gear has an operating efficiency of 98-99%. The pinion is made from a harder material than the wheel. A gear pair should be selected to have the highest number of teeth consistent with a suitable safety margin in strength and wear. The minimum number of teeth on a gear with a normal pressure angle of 20 degrees is 18.

It may not be possible to achieve target speed ratios precisely because many sizes will not be readily available 'off-the-shelf'. The preferred standard gear teeth numbers are:

12 13 14 15 16 18 20 22 24 25 28 30
32 34 38 40 45 50 54 60 64 70 72 75
80 84 90 96 100 120 140 150 180 200 220 250

The range of available sizes will depend upon the gear module (m). Generally the lower the module, the greater the range of gear teeth sizes available.

The preferred standard module sizes are: 0. 5, 0.8, 1, 1.2 5, 1.5, 2, 2.5, 3, 4, 5, 6

Large speed ratios are achieved in several stages, choosing the number of teeth for each gear so that the overall reduction is as close to target as is possible.

Choose the Face Width

The face width should be reasonably proportioned to other gear dimensions. If a tooth is too wide it may bend excessively across its width, if it is too narrow then an uneconomically large diameter must be provided to compensate for lack of width. Proportions may be expressed as:-

$w = \beta m$ where, usually, $9 \leq \beta \leq 15$ for economic gears -

These limits should not be regarded as inviolable, but costs should be expected to escalate if they are exceeded.

Material Selection

Choose a material for each gear that has the required strength using the catalogue or the following table of strengths and allowable stresses:-

Material, grade and condition		Tensile strength, Ts (MPa)	Allowable stresses	
			Bending, Tb (MPa)	Surface, st_s(MPa)
Nylon	-	65 (20°C) 36 (100°C)	12	-
Tuf nol	-	110	31	4
Cast Iron	GRI 7	247	62	10
Aluminium Alloy	HR1 5N	-	90	3
Phosphor Bronze	PB2	185	48	5
Steels				
EN 8 - 08OM40	-	540	131	9
EN 8 - 08OM40	Induction Hardened	540	117	19
EN 24 - 817M40	-	772	221	21
EN 24 - 817M40	Induction Hardened	772	183	34
EN 32 - 045M10	-	494	117	10
EN 32 - 045M10	Case Hardened	494	276	63
EN 36 - 655M13	Case Hardened	849	345	76
EN 58AM - 30S21	-	540	138	12

If it is necessary to design a gearbox from scratch, the design process in selecting the gear size is not complicated - the various design formulae have all been developed over time and are available in the relevant standards. However significant effort, judgment and expertise is required in designing the whole system including the gears, shafts, bearings, gearbox, and lubrication. For the same duty many different gear options are available for the type of gear, the materials and the quality.

GEAR COSTS

Generally gear costs increase with module size (i.e. tooth size and hence gear diameter) and gear type (due to manufacturing complexity). Typically a helical gear with a metric module of, say, 3 will be about two to three times more expensive than one, with the same number of teeth, with a module of 2. Also, spur gears will be less expensive than comparably sized helical gears, and in turn helical gears will cost less than internal, double helical and skew helical gear types. In addition, it should be remembered, that the larger diameter gears have larger bore sizes and so require larger shafts and possibly bearings also - this adds further to the overall cost. When selecting gears, after establishing suitable alternatives from technical considerations, it is usual to choose the lowest cost arrangement.

STANDARDS

- AGMA 2001-C95 or AGMA-2101-C95 Fundamental Rating factors and Calculation Methods for involute Spur Gear and Helical Gear Teeth
- ISO 6336-1:1996...Calculation of load capacity of spur and helical gears. Basic principles, introduction and general influence factors
- ISO 6336-2:1996...Calculation of load capacity of spur and helical gears. Calculation of surface durability (pitting)
- ISO 6336-3:1996...Calculation of load capacity of spur and helical gears. Calculation of tooth bending strength
- ISO 6336-5:2003...Calculation of load capacity of spur and helical gears. Strength and quality of materials

DEFINITIONS AND TERMINOLOGY OF PARALLEL AXIS GEAR

1) **Active Profile**- that part of the gear tooth which actually comes in contact with the profile of its mating tooth along the line of action.

2) **Addendum**- The height of the tooth above the pitch circle diameter.

3) **Arc of Action**- the arc of the pitch circle through which a tooth travels from the time it first makes contact with a mating tooth until contact with mating tooth ceases.

4) **Arc of Approach**- the arc of the pitch circle through which a tooth travels from the time it first makes contact with a mating tooth until it is in contact at the pitch point.

5) **Arc of Recess –** the arc through which the tooth moves from the time when the contact is at the pitch point until it ceases to be in contact with its mating tooth.

6) **Arc Space Width-** see Circular Space Width

7) **Arc Tooth Thickness –** see Circular Thickness

8) **Axle** - A round, hollow metal bar on which the rear wheels are mounted on.

9) **Axial Load Bearing** - A bearing in which the load acts in the direction of the axis of rotation.

10) **Axial Pitch**- the distance in an axial plane surface between corresponding adjacent tooth profiles.

11) **Axial Plane-** (of a pair of gears) the plane that contains the two axes. In a single gear, an axial plane may be any plane containing the axis and a point of its diameter.

12) **Axial Pressure Angle** - Is the angle between the tangent to the tooth profile in an axial plane at the pitch surface and a line perpendicular to the axis.

13) **Backlash**- the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles.

14) **Backlash Variation**- the difference between the maximum and minimum backlash occurring in a whole revolution of the larger of a pair of mating gears.

15) **Base Circular Thickness**- the length of arc on the base circle between two involute curves forming the profiles of a tooth.

16) **Base Cylinder**- the cylinder of the same diameter as the base circle.

17) **Base Lead Angle**- the lead angle on a base cylinder.

18) **Base Diameter**- the diameter of the circle from which the involute is generated.

19) **Base Helix Angle**- the helix angle on the base cylinder.

20) **Base Pitch**- (normal to involute) the circular pitch taken in the circumference of the base circle, and also the distance along the line of action between two successive and corresponding involute profiles.

21) **Base Radius**- the radius of the circle from which the involute is generated.

22) **Basic Rack-** For every pair of conjugate involute profiles there is a basic rack. This basic rack is the profile of the conjugate gear of infinite pitch radius.

23) **Bevel gears** - Are gears of conical form designed to operate on intersecting axes.

24) **Bottom Land**- the surface of the gear between the flanks of adjacent teeth.

25) **Center Distance**- The distance between axes of two gears in mesh.

26) **Chordal Addendum**- (normal) the perpendicular distance from the normal thickness chord to the top of the tooth.

27) **Chordal Thickness**- the length of the chord subtending the circular thickness arc.

28) **Circular Pitch**- the distance along the pitch circle or pitch line between corresponding profiles of adjacent teeth.

29) **Circular Space Width-** internal spline specifications use circular space width, rather than circular tooth thickness dimensions. Circular space width is equal to circular pitch minus circular tooth thickness.

30) **Circular Thickness**- the length of the arc on the pitch circle between the two sides of a gear tooth.

31) **Clearance**- the difference between the dedendum of one gear and the addendum of the mating gear.

32) **Cogwheel** - A wheel with a rim notched into teeth, which meshes with those of another wheel or a rack to transmit or receive motion.

33) **Common tangent** - The line tangent to the pitch circle at the pitch point.

34) **Composite Action**- the variation in center distance when two gears are rolled in tight mesh.

35) **Conjugate Curves-** gear teeth are a series of cam surfaces that act on similar surfaces of the mating gear. Curves that act on each other with a smooth driving action and with a constant driving ratio are called *conjugate curves*.

36) **Conjugate Action-** a smooth driving action that produces a constant angular velocity in the driven member.

37) **Contact Ratio**- the ratio of the arc of action to the circular pitch.

38) **Contact Ratio Face**- the ratio of the face advance to the circular pitch.

39) **Contact Ratio Total-** the ratio of the sum of the arc of action and the face advance to the circular pitch.

40) **Crowned Teeth**- teeth having surfaces modified in the lengthwise direction; in gears, to prevent contact at their ends. In splined couplings crowning accommodates slight angular misalignments.

41) **Dedendum**- the depth of the tooth below the pitch circle diameter.

42) **Diametral Pitch**- The diametral pitch (DP) is, by definition, the number of teeth (N) divided by the pitch diameter (D) i.e. DP=N/D.

43) **Differential gear** - A bevel gear that permits rotation of two shafts at different speeds.

44) **Double-Helical Gear**- a gear of cylindrical form with two sections of teeth, one right hand and the other left hand that engage simultaneously with the teeth of a similarly designed mating gear.

45) **Effective or (Active) Face Width**- the width of face that actually comes into contact with a mating gear.

46) **Equal-Addendum Teeth**- teeth of two engaging gears having same addendum.

47) **External Gear**- a gear with the teeth formed on the outer surface of a cylinder or cone.

48) **Face Advance**- the distance on the pitch circle that a gear tooth travels fromm the time pitch point contact is made at one end of the tooth until pitch point contact is made at the other end.

49) **Face Contact Ratio**- the contact ratio in an axial plane, or the ratio of the face width to the axial pitch. For bevel and hypoid gears, it is the ratio of face advance to circular pitch.

50) **Face gear** - It is a gear with cogs mortised into its face, usually in conjugation with a lantern pinion.

51) **Face width** - Is the width of the pitch surface.

52) **Face Width, Effective-** the width of the face that may actually come in contact with mating teeth.

53) **Face Width, Total**- (for double helical gears) the width of the pitch surface containing both of the helices and the groove width.

54) **Fillet Curve**- the concave portion of the tooth profile where it joins the bottom of the tooth space.

55) **Fillet Radius**- the radius of a circular arc approximating the fillet curve.

56) **Full Depth Teeth**- teeth in which the working depth equals 2.000 divided by normal diametral pitch.

57) **Gears** - It is a toothed machine part that meshes with another toothed part to transmit motion or to change speed or direction.

58) **Gear box** - The manual transmission on a shifter kart.

59) **Gear Blank-** the work piece used for the manufacture of a gear, prior to machining the gear teeth.

60) **Gear Center**- the center of the pitch circle.

61) **Gear ratio** - It is the ratio between number of teeth of the meshing gears.

62) **Gear train** - A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed.

63) **Groove Depth**- the depth of the clearance groove between helices of double helical gears.

64) **Groove Width-** the width of the clearance groove between helices of double helical gears.

65) **Helical Gear**- a cylindrical gear with helical teeth.

66) **Helical Rack**- a rack having teeth that are oblique to the direction of motion.

67) **Helix Angle**- the angle between a tangent to the helix and an element of the cylinder. Unless otherwise specified, the pitch helix is referred to.

68) **Herringbone Gears –** a type of double helical gear, having no clearance groove.

69) **Hub Diameter-** the diameter of the central part of the gear body surrounding the bore and extending beyond the web, spokes, or body.

70) **Hub Extension**- the distance that the hub extends beyond the face of the gear body.

71) **Idler gear** - A gear wheel placed between two other gears to transmit motion from one to the other.

72) **Inside Cylinder-** the surfaces that coincide with the tops of teeth of an internal cylindrical gear.

73) **Interference**- the contact between mating teeth at some other point than along the line of action.

74) **Internal gears** - Gears that have cylindrical pith surface with teeth parallel to the axis.

75) **Internal Diameter**- the diameter of that circle which contains the tops of teeth of an internal gear.

76) **Involute** - Is the curve described by the end of a line that is unwound from the circumference of a circle.

77) **Involute Curve-** the curve that is described by the end of a line that is unwound from the circumference of a circle. The circle from which the line is unwound is called the *base circle.*

78) **Involute Polar Angle**- the angle between a radius vector to a point on an involute curve and a radial line to the point where the curve touches the base circle.

79) **Involute Roll Angle**- an angle whose arc on the base circle of radius unity equals the tangent of the pressure angle at a selected point on the involute.

80) **Involute Teeth of Gears**- (helical gears and worms) are those in which the active portion of the profile in the transverse plane is the involute of a circle.

81) **Lead**- the axial advance of a helix for one complete turn, as in the threads of cylindrical worms and teeth of helical gears.

82) **Lead Angle**- the angle between a tangent to the pitch helix and a plane of rotation.

83) **Length of Action**- the distance on an involute line of action through which the point of contact moves during the action of the tooth profiles.

84) **Line of action** - Is that portion of the common tangent to the base circles along which contact between the mating involute teeth occurs.

85) **Line of Centers**- the straight line through the center of tangent pitch circles.

86) **Line of Contact**- the line or curve along which two tooth surfaces are tangent to each other.

87) **Long-and-Short Addendum Teeth**- the teeth of engaging gears (on a standard designed center distance), one of which has a long addendum and the other a short addendum.

88) **Miter gear** - Bevel gears put together with equal numbers of teeth and axes that are usually at right angles.

89) **Modified Addendum Teeth**- the teeth of engaging gears, one or both of which have nonstandard addendum.

90) **Modified Contact Ratio**- the contact ratio of modified tooth surfaces.

91) **Module**- (metric) the ratio of the pitch diameter in millimeters to the number of teeth. m=D/N

92) **Normal Base Pitch**- Distance on a normal base helix between corresponding involutes of adjacent teeth.

93) **Normal Chordal Addendum**- Chordal addendum in a normal plane.

94) **Normal Choral Thickness**- the length of the normal thickness chord between pitch line elements of a tooth.

95) **Normal Circular Thickness**- the circular tooth thickness in the normal plane. In helical gears, it is an arc of the normal helix.

96) **Normal Circular Pitch**- (normal to helix) the shortest distance on the pitch surface between corresponding pitch line elements of adjacent teeth.

97) **Normal Diametrical Pitch**- the diametrical pitch corresponding to the normal circular pitch and calculated in the normal plane.

98) **Normal Helix**- the helix on a pitch cylinder normal to the pitch helix.

99) **Normal Plane-** the plane perpendicular to a given straight line or to a tangent to a curved line.

100) **Normal Pressure Angle**- the pressure angle in plane normal to the pitch line element.

101) **Normal Profile Angle**- the profile angle in a normal plane of a helical or spiral tooth.

102) Normal Tooth Profile- the outline formed by the intersection of a tooth surface and a plane perpendicular to the pitch line element.

103) **Outside Diameter of Gear**- The diameter of the circle that contains the tops of the teeth of an external gear.

104) **Operating Pitch Diameters**- the diameter of the circle on a gear which is proportional to the gear ratio and the actual center distance at which the gear pair will operate.

105) **Operating Pressure Angle-** determined by the center distance at which the gears operate; it is the pressure angle at the operating pitch diameter.

106) **Outside Cylinder**- the surface that coincides with the tops of the teeth of an external cylindrical gear.

107) **Outside Helix**- the helix formed by the intersection of a tooth surface and the outside cylinder.

108) **Outside Helix Angle**- the helix angle on the outside cylinder.

109) **Outside Lead Angle**- the lead angle on the outside cylinder.

110) **Outside Radius of Gear or Pinion**- the radius of the circle which contains the tops of the teeth of external gears.

111) **Path of contact** - The path traced by the contact point of a pair of tooth profiles.

112) **Pawl** - A mechanical device allowing rotation in only one direction.

113) **Pinion**- a gear with a small number of teeth. Of two gears that run together, the one with the smaller number of teeth is called the pinion.

114) **Pitch, Circular**- the distance on the circumference of the pitch circle between corresponding points of adjacent teeth.

115) **Pitch Circle**- (see pitch diameter and pitch radius) the circle through the pitch point having its center on the axis of the gear.

116) **Pitch Cylinder**- the imaginary cylinder in a gear that rolls without slipping on a pitch cylinder or pitch plane of another gear.

117) **Pitch, Diametral**- (in plane of rotation) the ratio of the number of teeth to the number of inches in the pitch diameter. It indicates the number of teeth in the gear for each inch of pitch diameter.

118) **Pitch Diameter (standard)** - the circle which intersects the involute at the point where the pressure angle is equal to the profile angle of the basic rack. In parallel axis gearing, the standard pitch diameter of a gear is determined by dividing the number of teeth in the gear by the transverse diametral pitch.

119) **Pitch Helix**- the helix formed by the intersection of the surface of a helical tooth or thread with the pitch cylinder.

120) **Pitch Helix Angle**- the angle between any helix and an element of its cylinder.

121) **Pitch Lead Angle**- the lead angle on a pitch cylinder.

122) **Pitch Radius of Gear or Pinion**- the radius of the pitch circle.

123) **Pitch Point**- the intersection between the axes of the line of centers and the line of action.

124) **Pitch Range**- the difference between the longest and the shortest pitches on a gear.

125) **Pitch Surface**- the surface of the pitch cylinder, which rolls with the surface of the mating member.

126) **Plane of Action**- a surface of action in involute parallel axis gears, tangent to their base cylinders.

127) **Planetary gears** - A device allowing several gears to orbit about others.

128) **Point of Contact**- the point at which two tooth profiles touch each other.

129) **Plane of Rotation**- any plane perpendicular to a gear axis.

130) **Pressure Angle**- the angle between a tooth profile and the radial line at its pitch point. The pressure angle is often described as the angle between the line of action and a line tangent to the pitch circle. Standard pressure angles are established in connection with standard geartooth proportions.

131) **Pressure angle** - The angle between the tooth profile at the pitch circle diameter and a radial line passing through the same point.

132) **Profile Angle-** the angle between a tangent to a tooth profile and the radius from the gear axis to the tangent point. In gear cutting tools, the angle between a cutting edge and some principal direction in the tool.

133) **Profile Control Diameter**- the diameter of the circle beyond which the tooth profile must conform to the specified involute curve.

134) **Profile Radius of Curvature**- the instantaneous radius of a tooth profile at a specified point.

135) **Protuberance-** tooth form modification on the gear-generating tool to remove material below the form diameter on the gear tooth to allow clearance for finishing by grinding or shaving.

136) **Rack, General**- a gear with teeth spaced along a straight line and suitable for straight-line motion.

137) **Rack gear** - It is a gear of infinite pitch radius into which a pinion meshes.

138) **Ratchet** - A toothed wheel or bar that catches and holds a pawl, which thus prevents backward movement.

139) **Right Hand Helical Gear**- a gear in which the teeth twist clockwise as they recede from an observer looking along the axis.

140) **Root Circle**- the circle containing the bottom of the tooth spaces.

141) **Root Cylinder**- the imaginary cylinder tangent to the bottoms of the tooth spaces in a cylindrical gear.

142) **Root Diameter of Gear, Worm, or Pinion**- the diameter of the circle which contains the roots of the teeth.

143) **Root Radius**- the radius of the root circle.

144) **Single-Helical Gears**- helical gears having teeth of only one hand on each gear.

145) **Space, Bottom**- a line joining two fillets of adjacent tooth profiles in the same plane.

146) **Spiral bevel gears** - Designed for an angle change of 90 degrees, where the two axes are concurrent and in the same plane.

147) **Sprockets** - Are teeth like projections arranged on a wheel rim to engage the links of a chain.

148) **Spur Gears**- gears that are cylindrical in form, with teeth that are straight and parallel and that operate on parallel axes.

149) **Spur Rack**- a rack with straight teeth that are at right angles to the direction of motion.

150) **Stub Teeth**- teeth having a working depth less than 2.000 divided by normal diametral pitch.

151) **Sun gear** - Small gearwheel that revolves around its own axis and also around a gear meshing with it.

152) **Surface of Action**- the imaginary surface in which contact occurs between two engaging tooth surfaces. It is the summation of the paths of action in all sections of the engaging teeth.

153) **Tangent Plane**- the plane tangent to a tooth surface at a point or line of contact.

154) **Tip Relief**- an arbitrary modification of a tooth profile whereby a small amount of material is removed near the tip of the gear tooth.

155) **Tolerance, Tooth Alignment-** the permissible amount of tooth alignment variation; values are normal to the tooth surface.

156) **Tolerance, Tooth Thickness-** the permissible amount of tooth thickness variation.

157) **Tooth Bearing**- that portion of the tooth surface which actually comes into contact.

158) **Tooth Chamfer**- the bevel between the end of a tooth and the tooth surface, to break the sharp edge.

159) **Tooth Face**- the surface between the pitch line element and the top of the tooth.

160) **Tooth Fillet**- the curved surface of the tooth flank joining it to the bottom land.

161) **Tooth Flank**- the surface between the pitch line element and the bottom land; it includes the fillet.

162) **Tooth Profile**- the outline in a designated plane of a tooth between the addendum and root circles.

163) **Tooth Surface**- the total area including the tooth face and the tooth flank.

164) **Tooth Thickness on Base Circle**- the distance on the base circle between involutes of the same tooth.

165) **Tooth-to-Tooth Composite Variation**- the greatest change in center distance while the gear being tested is rotated through any angle of 360°/N during double flank composite action testing.

166) **Tooth Top**- a line joining the outer ends of two adjacent tooth profiles in the same place; in internal gearing it is the inner ends of the teeth.

167) **Top Land**- the surface on the top of the tooth.

168) **Total Composite Action-** the total amount of composite action for an entire gear.

169) **Total Contact Ratio-** the sum of the transverse contact ratio and the face contact ratio.

170) **Total Face Width-** the actual dimension of a gear blank that exceeds the effective face width; in double-helical gears, the total face width includes any distance separating right-hand and left-hand helices.

171) **Transverse Circular Thickness**- the circular tooth thickness in the plane of rotation.

172) **Transverse Contact Ratio**- the ratio of the arc of action to the transverse circular pitch.

173) **Transverse Diametral Pitch**- the ratio of the number of teeth to the number of inches in the transverse pitch diameter.

174) **Transverse Pitch**- the distance between corresponding pitch line elements of adjacent teeth in the plane of rotation.

175) **Transverse Plane**- perpendicular to the axial plane and to the pitch plane. In gears with parallel axes, the transverse plane and plane of rotation coincide.

176) **Transverse Pressure Angle**- the pressure angle in the plane of rotation.

177) **Transverse Profile Angle**- the profile angle in a transverse plane.

178) **Trochoid-** a curve which is traced on a rotating plane by a designated point on a second plane. This is commonly the form of a generated root fillet.

179) **Undercut**- a condition in generated gear teeth when part of the fillet curve lies inside of a line drawn tangent to the true involute form at its lowest point. Undercut may be deliberately introduced to facilitate finishing operations.

180) **Velocity ratio** - The ratio of the number of revolutions of the driving gear to the number of revolutions of the driven gear, in a unit of time.

181) **Whole Depth**- (total depth) the radial distance between the outside circle and the root circle.

182) **Working Depth-** the greatest depth to which a tooth of one gear extends into the tooth space of the mating gear.

183) **Worm gear** - It is a gea with one or more teeth in the form of screwed threads.

184) **Zerol bevel gears** - Is a bevel gear with intersecting shafts.

185) Zone of Action- the rectangular area in a plane of action limited by the length of action and the face width.

For more detailed coverage of this subject, consult ANSI/AGMA Standard 1012-F90; Gear Nomenclature, Definitions with Terms and Symbols